In chapter 7, “The Mathematics of Networks,” you will learn how to find optimal networks connecting a set of points. A **network** is a connected graph with vertices and edges. For example, the vertices in a railroad network are the train stations and the edges are the railroad lines between stations. In an electric power network, the vertices are the power stations and the edges are power lines connecting the stations. An **optimal** network is the shortest network.

**Definitions**

**Network** – a connected graph  
**Tree** – a network (connected graph) with no circuits  
**Spanning Tree** – a “subgraph” of a network (connected graph) that (1) connects all the vertices of a network and (2) has no circuits  
**Minimum Spanning Tree** – in a weighted network, the spanning tree with the least total weight

**Exercise 1.** Use the definitions above determine if the graphs are (1) networks and (2) trees.

1.  
2.  
3.  
4.  
5.  
6.  
7.  
8.  
9.
Properties of Trees

Exercise 2. For Figures 1 through 4, determine (1) if the graph is a tree and (2) the number of paths between vertex A and vertex I.

(1) tree? _______ (1) tree? _______ (1) tree? _______ (1) tree? _______
(2) paths _______ (2) paths _______ (2) paths _______ (2) paths _______

Exercise 3. For Figures 1 through 4, identify every edge that is not a bridge. If every edge is a bridge, write, “Every edge is a bridge.”

Figure 1 ______________________________________________________________________
Figure 2 ______________________________________________________________________
Figure 3 ______________________________________________________________________
Figure 4 ______________________________________________________________________

Exercise 4. For the trees in Figures 5 through 8, identify (1) the number of vertices and (2) the number of edges.

(1) vertices _____ (1) vertices _____ (1) vertices _____ (1) vertices _____
(2) edges _______ (2) edges _______ (2) edges _______ (2) edges _______

Based on your observations in Exercises 2 through 4, complete the statements below that comprise the properties of trees.

Property 1: In a tree, there is only _______ path(s) joining any two vertices.

Property 2: In a tree, every edge _______________ (is or is not) a bridge.

Property 3: A tree with $N$ vertices has _____________ edges.