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Chapter 2: Explore Proportional and Linear Relationships (3 weeks)

Utah Core Standard(s):

- Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has a greater speed.* (8.EE.5)
- Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*. (8.EE.6)

Academic Vocabulary: slope, unit rate, rate of change, proportional constant, *m*(slope), translation, dilation, similar triangles, *b*(y-intercept),linear, right triangle, origin, rise, run,

Chapter Overview:

In this chapter students recognize equations for proportions ($\frac{y}{x} = m$ or $y = mx$) as special linear equations $y = mx + b$), understanding that the constant of proportionality or unit rate (m) is the slope, and the graphs are linear through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount *A*, the output or y-coordinate changes by the amount *m* times *A*.

In brief, each section addresses the following.

Section 1

- Reviews proportional relationships from 6^{th} and 7^{th} grade
- Introduces equations (*y*=*mx*) and graphs(a straight line through the origin) of proportional relationships
- Investigates the constant of proportionality or unit rate, and relates it to slope
- Compares and contrasts proportional relationships represented in different ways

Section 2

- Looks at slope in a contextual situation
- Establishes slope as $m = \frac{y}{x}$ or $\frac{rise}{run}$
- Integrates the fact that dilations produce similar triangles and that slope can be calculated from any two distinct points on a line.
- Derives the Slope Formula

Section 3

- Recognizes that a proportional relationship is special linear relationship
- Relates a proportional equation *(y=mx)* to a linear equation *(y=mx+b)* through a translation of a line.

Connections to Content:
Prior Knowledge: This chapter relies heavily on students knowledge about ratios and proportional relationships from 7th grade. Students should come with an understanding of computing unit rates. In addition they need to be able to recognize and represent proportional relationships from a story, graph, table, or equation. As well as identify the constant of proportionality amongst these representations.
Future Knowledge: After this chapter students begin to work with linear relationships and functions. Students use their knowledge of slope and proportionality to represent linear functions in a variety of ways. They begin to construct functions and use the rate of change or slope to do so.

MATHEMATICAL PRACTICE STANDARDS (emphasized):

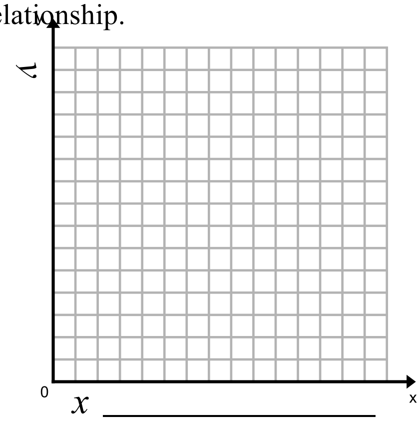
Make sense of problems and persevere in solving them

Gourmet jellybeans cost $9 for 2 pounds.

a. Complete the table.

Pounds	.5		3	4		8	10		20
Total Cost		$9			$27				

b. Label and graph axes. Graph the relationship.



c. What is the unit rate?

d. Write a sentence with correct units to describe the rate of change.

e. What is the slope of the line?

f. Write an equation to find the cost for any amount of jellybeans.

g. Why is the data graphed only in the first quadrant?

As students approach this problem they are given some real world data and asked to graph and analyze it. They must make conjectures about the slope of the line and understand the correspondences between the table, graph, and equation. The final question asks students to conceptualize the problem by having them explain why only the first quadrant was used.

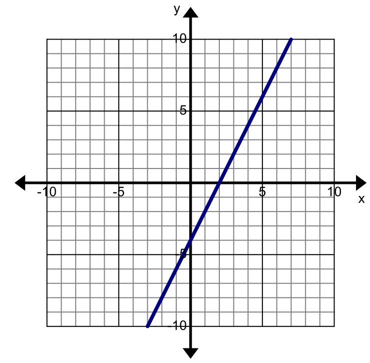
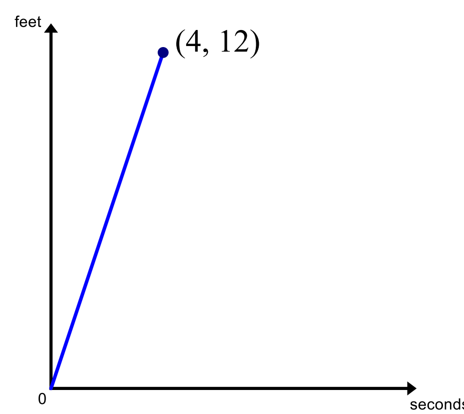
Reason abstractly and quantitatively

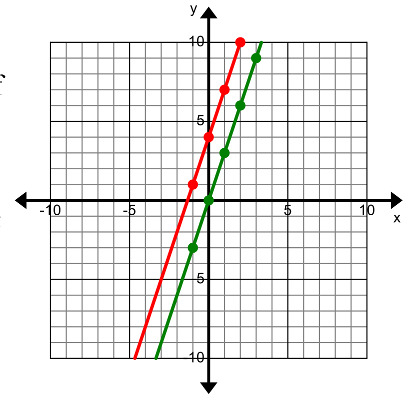
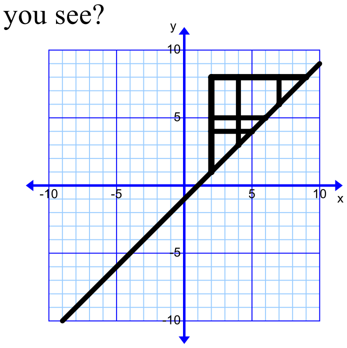
Graphing points can be time-consuming. Develop a procedure for calculating the slope without graphing each point. Explain your procedure below. Show that it works for problems 1-4 above.

Discuss the methods for calculating slope without using right triangles on a graph. Write what you think about the methods.

Now discuss this formula:. $m = \frac{y_2 - y_1}{x_2 - x_1}$ What does it mean? How does it work?

By examining how the rise and run is found amongst a variety of points students begin understand that the rise is the difference of the y values and the run is the difference of the x values. They must abstract the given information and represent it symbolically as they develop and analyze the slope formula.

	Construct viable arguments and critique the reasoning of others	On the line to the right choose any two points that fall on the line. (To make your examination easier choose two points that fall on an intersection of the gridlines).  From the two points create a right triangle, the line itself will be the hypotenuse and the legs will extend from the two points and meet at a right angle. Compare the points that you choose and your triangle with someone in your class. Discuss the following; Did you both choose the same points? How are your triangles the same? How are your triangles different? What relationship exists between your triangles? *Upon comparing their triangle with a class member students begin to discover that any right triangle that is constructed is a similar triangle. By talking with one another they can analyze different triangles and discuss the similarity that exists between them. This also gives students the opportunity to help one another learn how to accurately construct the right triangle on the graph used to find slope. They can also begin to make conjectures about how slope have be found from any two points on the line.*
	Model with mathematics	a. Create your own story that shows a proportional relationship. b. Complete a table and graph to represent this situation. Be sure to label the axes of your graph. c. Write an equation that represents your proportional relationship. *This question asks students to not only create their own proportional relationship but to model it with a table, graph, and equation. Students show their understanding of how a proportional relationship is shown in several representations.*
	Attend to precision	The graph below shows the distance a cat is from his bowl of milk over time. Which sentence is a good match for the graph?  A. The cat was 12 feet away from the milk and ran toward it reaching it after 4 seconds. B. The cat was 4 feet away from the milk and ran toward it reaching it after 12 seconds. C. The cat ran away from the milk at a rate of 3 feet per second. D. The cat ran away from the milk at a rate of 4 feet per second. E. The cat was 12 feet away from the milk and ran away from it at a rate of 4 feet per second. *Upon examining the graph students must attend to precision as they discuss the order pair (4,12) and analyze exactly what it is telling us about the cat. Students often confuse the given information with rate of change and what unit each number represents. Also they must communicate what the direction of the line is telling us. Later on they are asked to make their own graphs and must use the correct units and labels to communicate their thinking,*

	Look for and make use of structure	What is the equation of the proportional relation that coincides with this shift.(Think about how the original equation would change based off of how you shifted the graph.  *In this problem students are analyzing the structure of a proporitonal equation and using a transformation to create a non-prorportional linaer equation. By looking at the existing line and equation they can dicern the pattern of identifying how many spaces the line was transformed and then applying that operation to the equation.*
	Use appropriate tools strategically	Your group task is to build a set of stairs and a handicap ramp at the side. Both the stairs and the ramp will begin at the same place (at ground level) and end at the height of 3 feet. Answer the following questions as you develop your design. • How many steps do you want or need? • How deep should each step be (we'll call this the run)? Why do you want this run depth? • How tall will each step be (we'll call this the rise)? Why do you want this rise height? • What is the total distance (total depth for all steps) you will need (at the base) for all of the stairs—this would be a measurement at ground level from stair/ramp start point to stair/ramp end point? Sketch the ramp (as viewed from the side) on graph paper below. Label and sketch the base and height, for example: Stair-base (in inches or feet) and Height (in inches or feet). *As students design a set of stairs and ramps they must use decide how they can use the tools (graph paper, ruler, pencil) provided them most efficiently. They will need generate a graph that displays their designs and use the graph as a tool to analyze the slope of the stairs and ramp.*
	Look for and express regularity in repeated reasoning	In each graph below, how many right triangles do you see? • Trace the triangles by color. • For each triangle write a ratio comparing the lengths of its legs or $\frac{height}{base}$. Then simplify the ratio $\frac{height}{base} = \underline{\qquad}$.  *In this series of problems students repeatedly find the height/base ratio of similar triangles and infer that slope can be calculated with the rise/run ratio by choosing any two points.. A general method for finding slope as rise/run is discovered.*

Section 2.1: Analyze Proportional Relationships

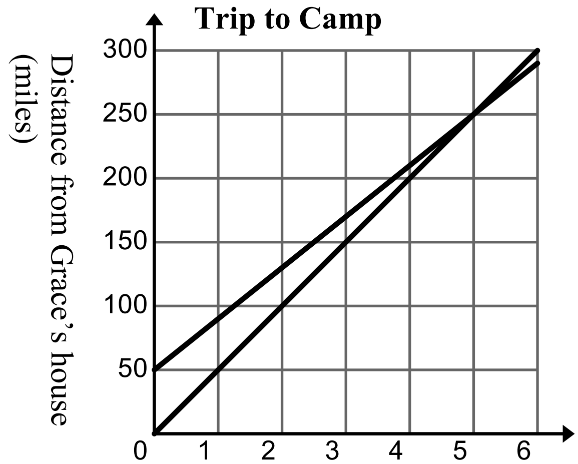
Section Overview:

The chapter begins by reviewing proportional relationships that were learned in 6th and 7th grade. The graph and equation ($y = mx$) of a proportional relationship is reviewed and the idea that the constant of proportionality or the unit rate is the slope ($m = \frac{y}{x}$)is established. Students then investigate the transition of the proportional constant or unit rate to slope, that is, if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount *m* times *A*. Finally proportional relationships that are represented in different ways are compared to one another.

Concepts and Skills to Master:
By the end of this section, students should be able to:
1. Find a proportional constant from a table of values, a story or context, and/or a graph/
2. Graph and write equations for a proportional relationship given a table, equation, or contextual situation.
3. Know that a proportional relationship is a linear graph that goes through the origin.
4. Understand that the proportional constant or unit rate is the ratio that relates the change in y values to the change in x values, it is the same thing as the slope of a line.
5. Compare proportional relationships represented in different ways.

2.1a Class Activity: Representions of Proportional Relationships

Grace and Kelly are both headed to summer camp. Camp is 250 miles from Grace's house. Kelly lives 50 miles closer to camp so her house is only 200 miles from camp. Grace and Kelly are so excited to see each other that they both want to arrive at camp at exactly the same time. They determine that if they both leave their house at 9 am and Grace drives at 50 mph and Kelly drives at 40 mph, they will both arrive at camp at the same time. The graph below represents this situation.



1. Label the graph to illustrate which line shows Grace's journey and which line shows Kelly's journey.

2. Analyze the graph and write down everything the graph is telling you in the space to the right of the graph.

3. In the tables below, let t = time in hours and let d = distance from Grace's house in miles.
   a. Start by completing the distance column for both girls.
   b. Then complete the column with the ratio $\frac{d}{t}$ for both girls.

Grace		
t	d	$\frac{d}{t}$
0		
1		
2		
3		
4		
5		

Kelly		
t	d	$\frac{d}{t}$
0		
1		
2		
3		
4		
5		

4. What do you notice about the ratio $\frac{d}{t}$ for Grace? What do you notice about the ratio $\frac{d}{t}$ for Kelly?

5. Using the information from the table, determine the distance for $t = 6$ for both girls. Explain how you came up with your answers.

6. Write an equation that represents each girl's distance d from Grace's house after t hours.

   Grace: ______________________________

   Kelly: ______________________________

7. In the case of Grace, distance and time are **proportionally related**. When two quantities are proportionally related, their ratio is constant. The constant is called the **proportional constant.** As you observed in the table above for Grace, the ratio $\frac{d}{t}$ is constant while for Kelly it is not. In the case of Kelly, distance and time are *not* proportionally related.

   What do you see in each of the following representations when two quantities are **proportionally related**?
   - In the context:
   - In the graph:
   - In the table:
   - In the equation:

8. Grace travels 50 mph. This means that she travels 50 miles in 1 hour. This is a unit rate. A **unit rate** describes how many units of the first quantity correspond to one unit of the second quantity. Go back to Grace's graph, table, and equation and illustrate where you see the unit rate of 50 mph.
   - In the context:
   - In the graph:
   - In the table:
   - In the equation:

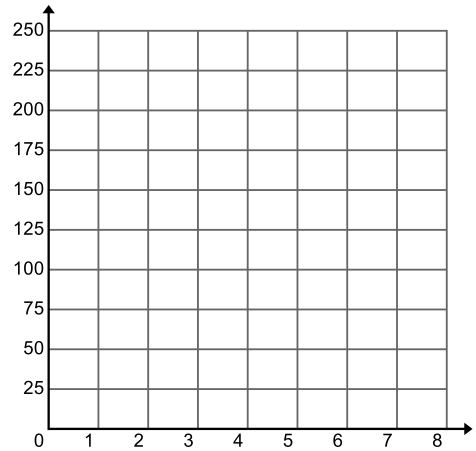
9. How are unit rate and the proportional constant related to each other?

Directions: In the following situations determine if the two quantities are in a proportional relationship.

10. Bubba's Body Shop charges \$25 per hour to fix a car.

a. Complete the table and graph to represent this situation. Be sure to label the axes of your graph.

Total Cost		
t (time in hours)	***C (cost in dollars)***	$\frac{C}{t}$
0		
1		
2		
3		
4		
5		



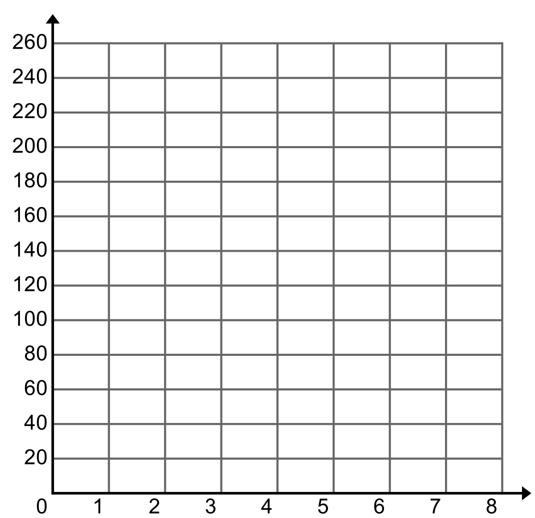
b. Write an equation for the cost *C* Bubba charges to fix a car for *h* hours.

c. Are cost and time proportionally related in this situation? Justify your answer using information from your table, graph, and equation.

11. Mike's Mechanics charges an initial fee of \$40 and then an additional \$20 per hour to fix your car.

a. Complete the table and graph to represent this situation. Be sure to label the axes of your graph.

Total Cost		
t (time in hours)	***C (cost in dollars)***	$\frac{C}{t}$
0		
1		
2		
3		
4		
5		

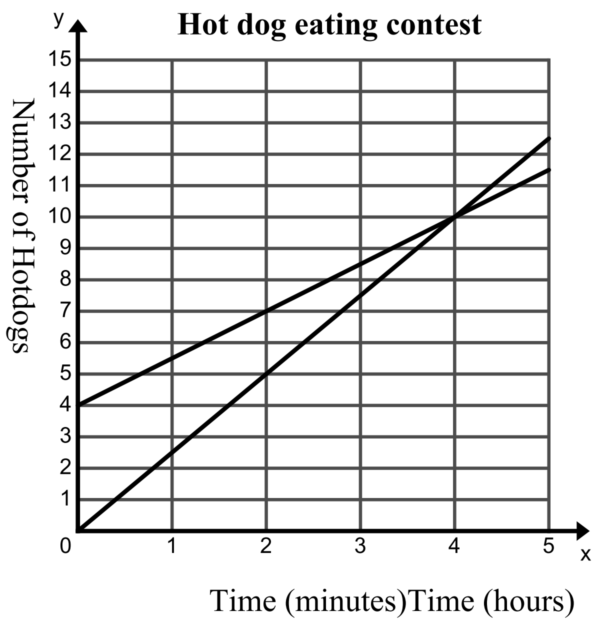


b. Write an equation for the cost *C* Mike charges to fix a car for *h* hours.

c. Are cost and time proportionally related in this situation? Justify your answer using information from your table, graph, and equation.

2.1a Homework: Representations of Proportional Relationships

During a 5 minute long hot dog eating contest Landon eats 5 hot dogs every 2 minutes. Nate eats 4 hot dogs before the competition even begins to stretch out his stomach and once the competition begins he eats 3 hot dogs every 2 minutes. The graph below represents this situation



1. Label the graph to illustrate which line shows Landon's eating and which line shows Nate's eating.

2. Analyze the graph and write down everything the graph is telling you in the space to the right of the graph.

3. In the tables below, let t = time in minutes and let h= number of total hotdogs consumed.
4.
   a. Start by completing the hotdog column for both boys.
   b. Then complete the column with the ratio $\frac{h}{t}$ for both girls.

Landon		
t	h	$\frac{h}{t}$
0		
1		
2		
3		
4		
5		

Nate		
t	h	$\frac{h}{t}$
0		
1		
2		
3		
4		
5		

5. What do you notice about the ratio $\frac{h}{t}$ for Landon? What do you notice about the ratio $\frac{h}{t}$ for Nate?

6. Using the information from the table, determine the number of hotdogs for $t = 6$ for both boys. Explain how you came up with your answers.

7. Write an equation that represents the number of hotdogs h for each boy after t minutes

   Landon: ______________________________

   Nate: ______________________________

8. Which person, Landon or Nate shows a proportional relationship? Explain how you know for each representation below

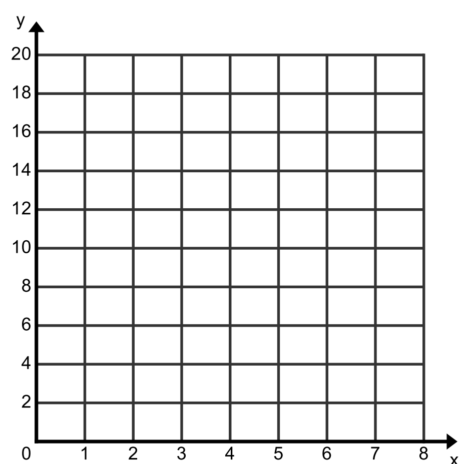
   - In the context:
   - In the graph:
   - In the table:
   - In the equation:

Directions: In the following situations determine if the two quantities are in a proportional relationship.

9. Sunset Splash and Spa charges an entrance fee of \$4.00 and then \$2.00 per hour.

   a. Complete the table and graph to represent this situation. Be sure to label the axes of your graph.

Total Cost		
t (time in hours)	***C (cost in dollars)***	$\boldsymbol{\frac{C}{t}}$
0		
1		
2		
3		
4		
5		



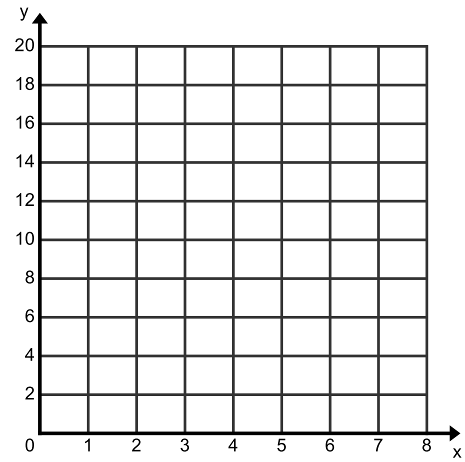
   b. Write an equation for the cost C that Sunset Splash and Spa charges to swim for h hours.

   c. Are cost and time proportionally related in this situation? Justify your answer using information from your table, graph, and equation.

10. Paradise Pool charges $3.00 per hour to swim at their pool

a. Complete the table and graph to represent this situation. Be sure to label the axes of your graph.

Total Cost		
t (time in hours)	*C (cost in dollars)*	$\frac{C}{t}$
0		
1		
2		
3		
4		
5		

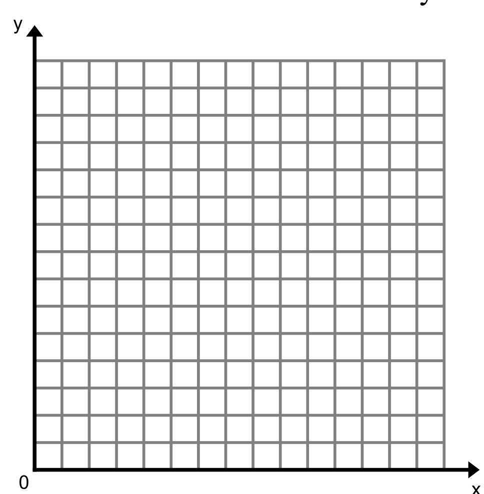


b. Write an equation for the cost *C* that Paradise Pool charges to swim for *h* hours.

c. Are cost and time proportionally related in this situation? Justify your answer using information from your table, graph, and equation.

11. a. Create your own story that shows a proportional relationship.

b. Complete a table and graph to represent this situation. Be sure to label the axes of your graph.



c. Write an equation that represents your proportional relationship.

2.1b Class Activity: Proportional or Not?

1. For each scenario below complete the table and determine if it shows a proportional relationship. If so state the proportional constant.

   a. Julie is picking teammates for her soccer team. She is asking 2 girls for every boy.

Boys	1	3		5	
Girls			8		

   b. A seed grows ½ and inch every 4 days.

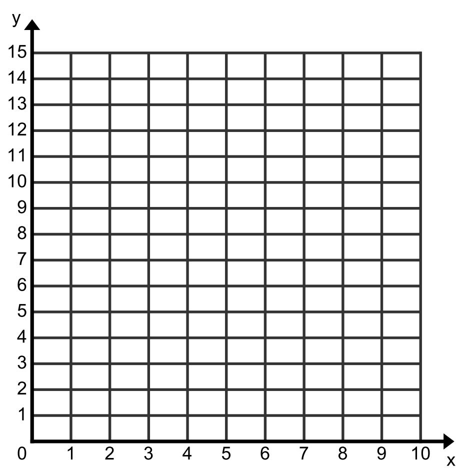
Days	0	4	8	12	16
Height (inches)	0			$1\frac{1}{2}$	

   c. The sum of two numbers is 0 (use *x* and *y* to represent the two numbers).

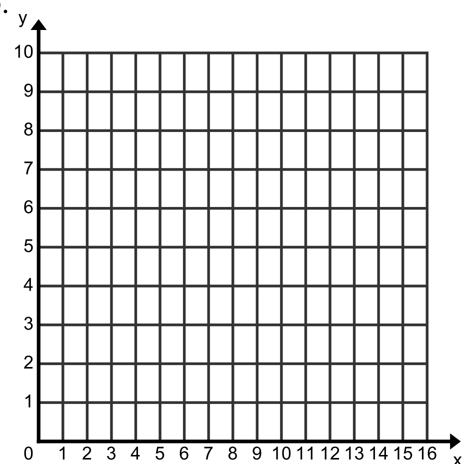
x	-2	-1	0	1	2
y					

2. Graph the data from each scenario in number 1 below.

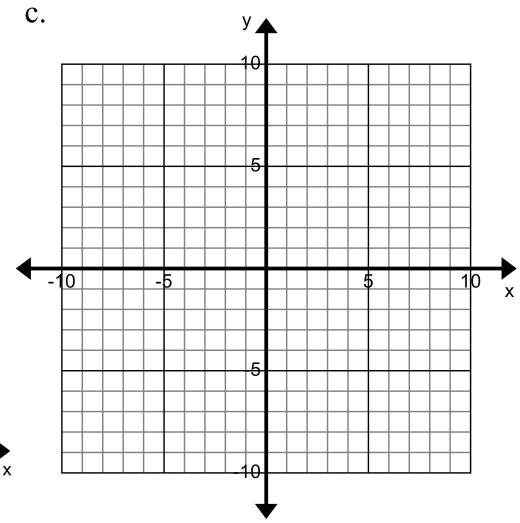
a.



b.



c.



d. What do the proportional graphs have in common?

e. What makes the non-proportional graph different?

3. Are the **relationships in the tables** proportional?

a.

x	*y*
1	2
2	6
3	10
4	14

b.

x	*y*
3	6
5	10
10	20
12	24
15	30

c.

x	*y*
2	1
4	2
6	3
8	4
10	5

d.

x	*y*
-2	-6
-0.5	-1.5
2	6
3	9
4	12

Justify why or why not.			
If proportional, write an equation that relates y to x : $y =$ _______. If NOT proportional write "NOT". Then Explain why not.			

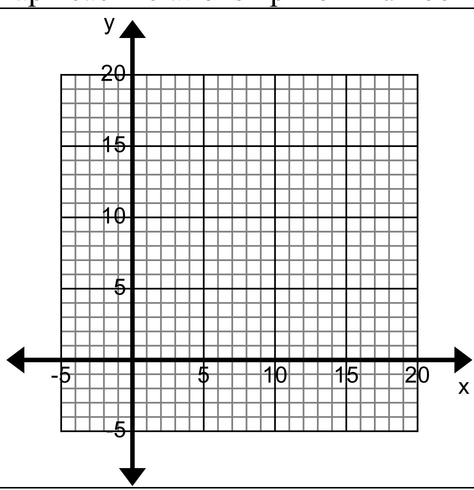
e. Analyze the equations from the proportional relationships, what do they have in common?

f. Predict what the graphs of the proportional relationships will look like.

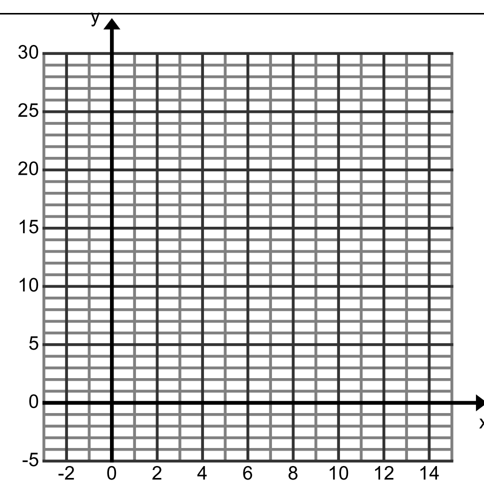
g. Predict what the graphs of the non-proportional relationships will look like.

4. Graph each relationship from number 4.

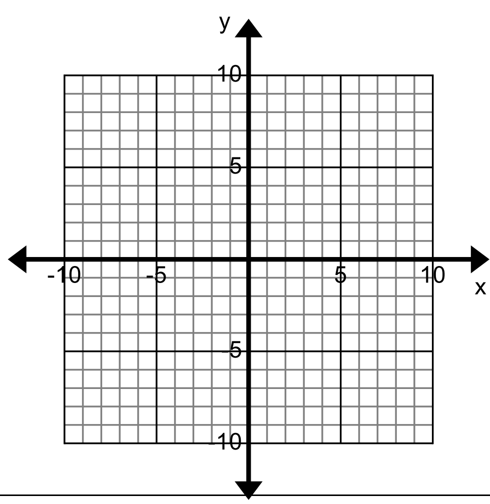
a.



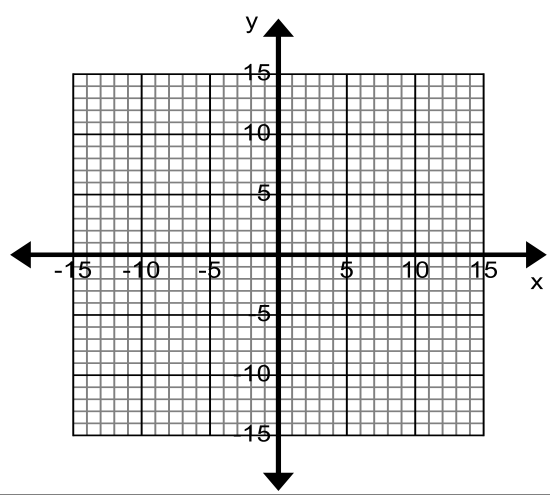
b.



c.



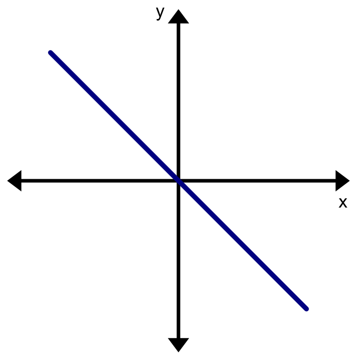
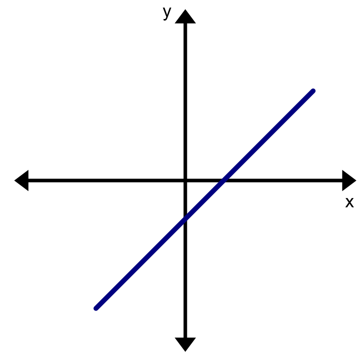
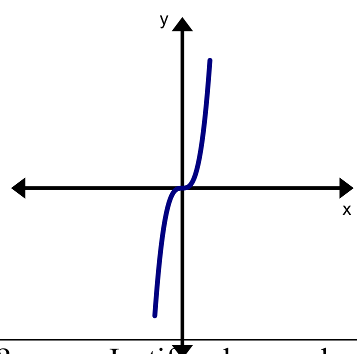
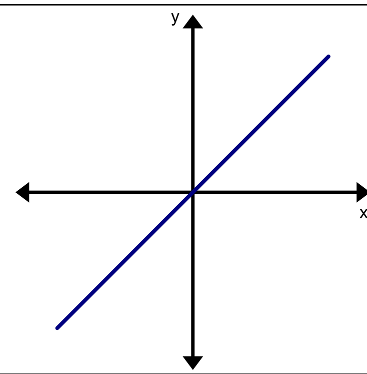
d.



5. Do the equations below show proportional relationships? Justify why or why not.

a. $y = 5x$　　b. $y = 2x + 5$　　c. $y = \frac{1}{2}x$　　d. $x + y = 4$　　e. $y = -3x$

6. Do the graphs below show proportional relationships?

a. 	**b.** 
Proportional? Justify why or why not.	Proportional? Justify why or why not.
c. 	**d.** 
Proportional? Justify why or why not.	Proportional? Justify why or why not.

7. Describe what the graph of a proportional relationship looks like.

8. Describe what an equation of a proportional relationship looks like.

9. Do the situations below involve proportional relationships?

a. During her first hour on Saturday, Fiona sold 8 sweaters at the Sweater Barn. After the second hour, she had sold 15 sweaters in all. After the third hour, she had sold 22 sweaters.

i. Is this a proportional relationship? How do you know?

ii. If it's proportional, what will the graph look like?

b. Toni ran in a marathon. After 1 hour, she had run 6 miles, after 2 hours, 12 miles, after 3 hours, 18 miles, after 4 hours, 24 miles.

i. Is this a proportional relationship? How do you know?

ii. If it's proportional, what will the table and graph look like?

2.1b Homework: Proportional or Not?

1. a. Let's say the ratio of pink flowers to purple flowers prepared for a wedding is $\frac{2}{3}$. Complete in the table for increasing numbers of pink and purple flowers as more vases are added to the order.

Pink Flowers	2	8		20	
Purple Flowers			15		45

   b. We know the relationship of pink flowers to purple flowers is proportional as more vases are ordered. How can you see this in the table?

2. Are the **relationships in the tables** proportional? Justify why or why not.

a.

x	y
1	3
2	6
3	9
4	12
5	15

b.

x	y
–4	–10
–2	–4
0	0
2	4
4	8

c.

x	y
3	1
6	2
9	3
12	4
15	5

d.

x	y
–1	–1.5
1	1.5
3	4.5
5	7.5
7	10.5

Justify why or why not.

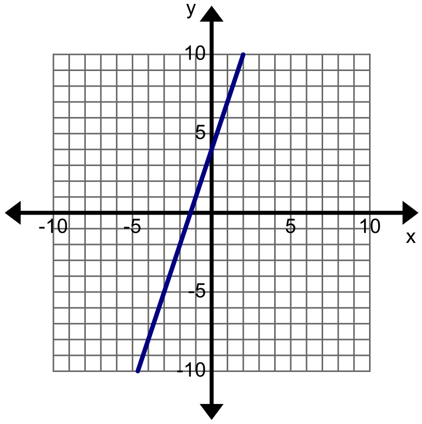
If proportional, write an equation : $y =$ _______. If NOT proportion write "NOT". Then Explain why not.

3. Do the equations below show proportional relationships**?** Justify why or why not.

a. $y = 2x$ b. $y = x - 1$ c. $y = -\frac{1}{4}x + 2$ d. $x + y = 0$ e. $y = 0.3x$

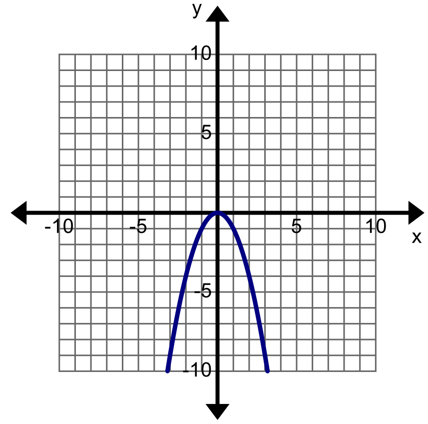
4. Do the graphs below show proportional relationships?

a.



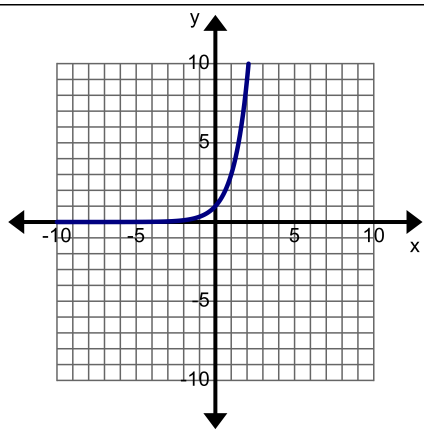
Proportional? Justify why or why not.

b.



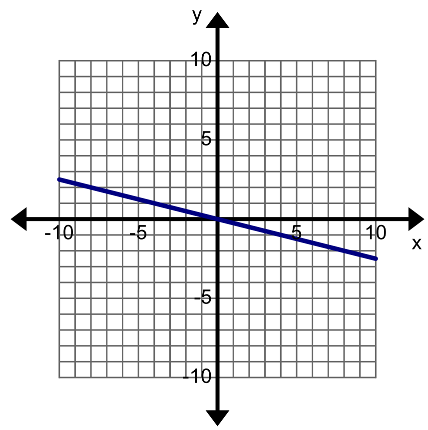
Proportional? Justify why or why not.

c.



Proportional? Justify why or why not.

d.



Proportional? Justify why or why not.

5. Do the situations below involve proportional relationships? Justify your reasoning.
   a. Tom planted a tree. The tree grew 3 inches in the first year. After 2 more years, the tree had grown a total of 8 inches.
      i. Is this a proportional relationship? How do you know?

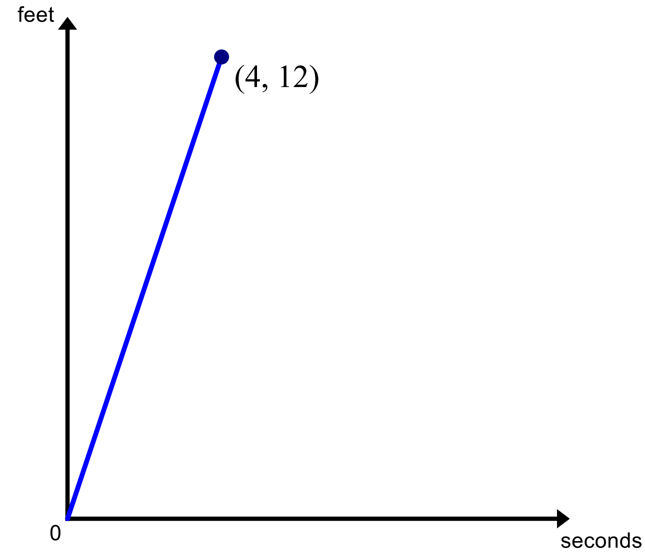
      ii. If it's proportional, write an equation to find the height for any number of years.

   b. Vanessa is mixing infant formula for her infant. For 3 ounces of water she needs to add 1 scoop of water.
      i. Is this a proportional relationship? How do you know?

      ii. If it's proportional, write an equation to find how many scoops of formula she needs to add for any number of ounces of water.

2.1c Class Activity: Proportional Constant, Unit Rate, and Slope

1. The graph below shows the distance a cat is from his bowl of milk over time. Which sentence is a good match for the graph?



A. The cat was 12 feet away from the milk and ran toward it reaching it after 4 seconds.

B. The cat was 4 feet away from the milk and ran toward it reaching it after 12 seconds.

C. The cat ran away from the milk at a rate of 3 feet per second.

D. The cat ran away from the milk at a rate of 4 feet per second.

E. The cat was 12 feet away from the milk and ran away from it at a rate of 4 feet per second.

2. Write everything you can say about the cat and the distance he is from the milk during this time.

3. Create a table at the right which also tells the story of the graph and your writing.

4. Is this a proportional relationship? If so what is the proportional constant?

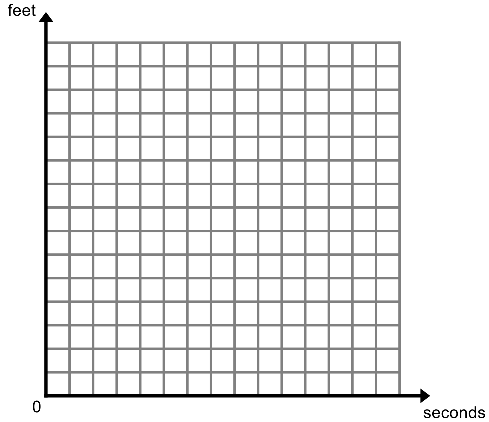
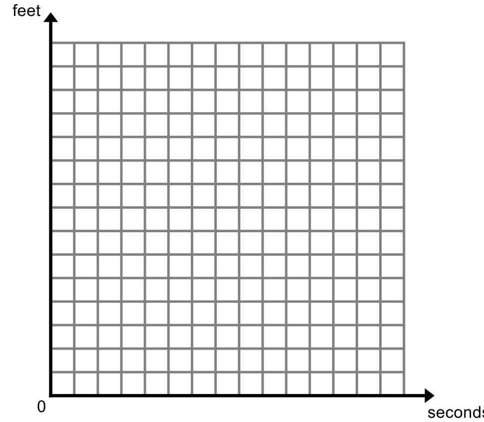
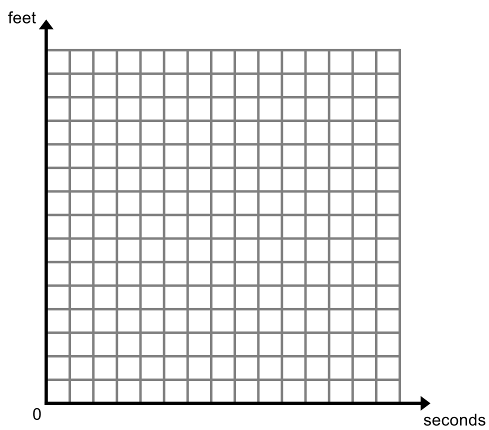
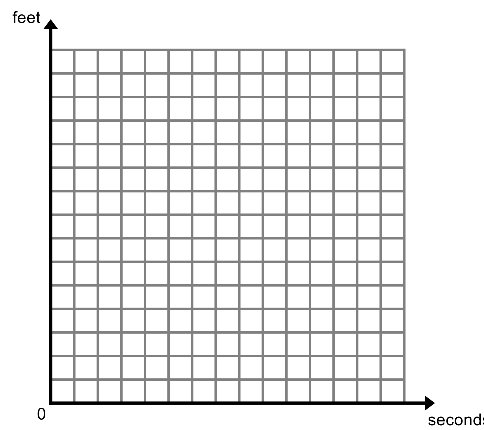
Remember that the unit rate describes the incremental change in one quantity for one additional unit of the other quantity or the rate of change. In this case it would be the distance(feet) the cat travels per second.

5. Find the unit rate in this story

6. Describe why you think the unit rate and the constant of proportionality are the same in this story.

7. Sketch a graph for the four stories from number 1 above which you didn't choose in the space provided. Label the graphs by letter to match the story.

a. Determine if these stories are proportional or not, if they are find the proportional constant.

 a. Proportional(yes or no): b. Unit Rate:	 a. Proportional(yes or no): b. Unit Rate:
 a. Proportional(yes or no): b. Unit Rate:	 a. Proportional(yes or no): b. Unit Rate:

Examine the relationships the graphs that are not proportional above. They do not have a constant of proportionality, however, the change in the variables between two points are in proportion. This change is called the **rate of change** or **unit rate**.

7b. Find the unit rate or rate of change for each relationship(write your answer in the table above).

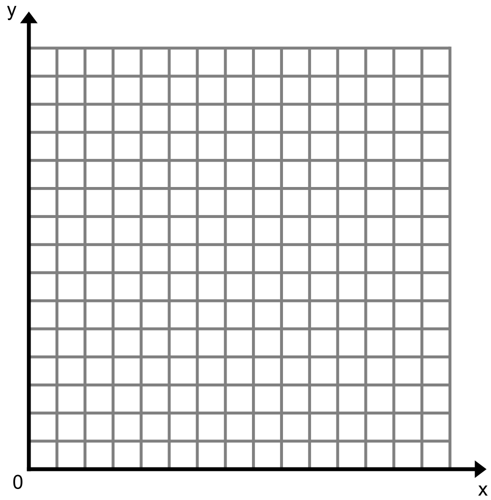
8. How does the unit rate/rate of change and proportional constant compare in relationships that are proportional?

9. How does the unit rate/rate of change and proportional constant compare in relationships that are not proportional?

10. Write a story and create a graph for these tables. Then determine if the relationship is proportional and find the unit rate.

a.

Time (seconds)	Distance (meters)
0	0
2	6
4	12
6	18
8	24
10	30
12	36



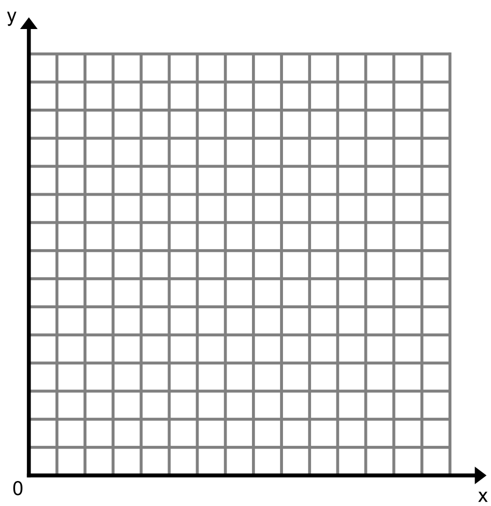
Story:

Proportional (yes or no):

Unit Rate:

b.

Time (seconds)	Distance (meters)
0	24
1	20
2	16
3	12
4	8
5	4
6	0



Story:

Proportional (yes or no):

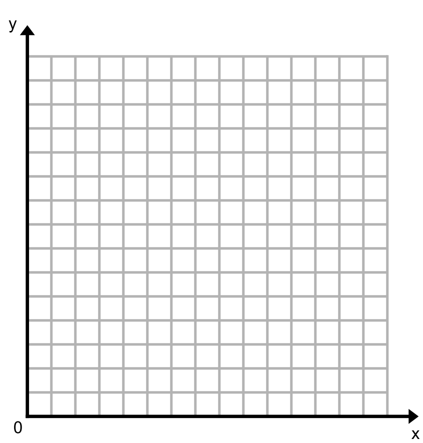
Unit Rate:

For each situation below, fill in tables and graphs and answer questions.

11. Bill's Burger Barn has a special deal of 4 hamburgers for $6

x (# of hamburgers)	y (cost)
0	
1	
2	
3	
4	

Sketch a graph. LABEL axes.



Is this a proportional relationship?

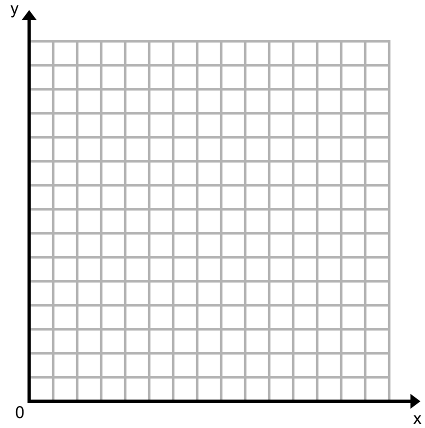
What is the unit rate (cost per hamburger):

Equation:

12. The bumble bee is five inches away from the flower and flies away from it. After 5 seconds the bumble bee is 10 feet away from the flower

x (# of feet)	y (distance)
0	
1	
2	
3	
4	

Sketch a graph. LABEL axes.



Is this a proportional relationship?

What is the unit rate:

Equation:

13. Jane took 16 pounds of aluminum cans to the recycling center and received $12.

a. What rate did the center pay for cans?

b. Is the unit rate a proportional constant? How do you know?

c. What will the graph look like using this rate?
(sketch if desired)

14. Billy has $30 in his piggy bank. Each week he takes out the same amount of money. After 5 weeks he has no money left in his piggy bank.

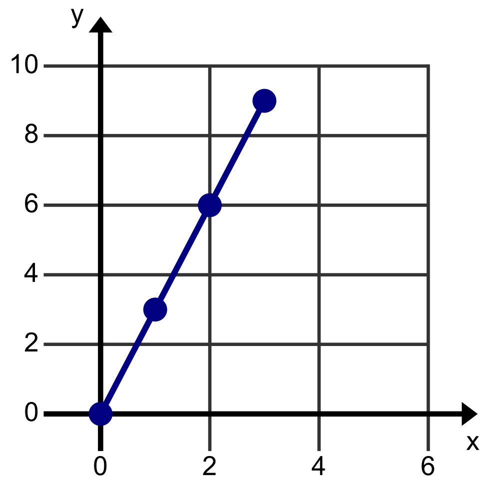
a. How much money does he take out of his piggy bank each week? How do you know?

b. Is the unit rate a proportional constant? Explain why or why not.

c. Describe what the graph would look like of this relationship.

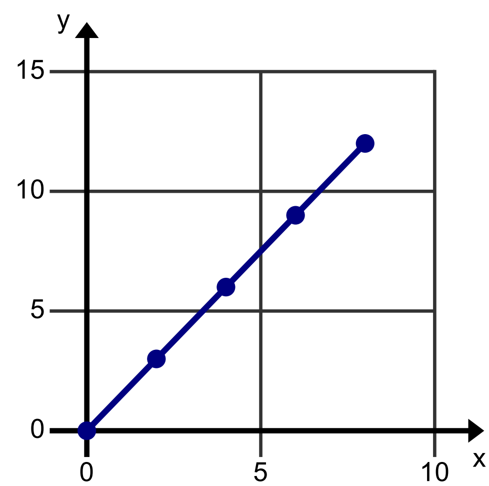
15. Examine the graphs below. If desired, give them a context to help you think (try miles per hour). Determine if the graphs show a proportional relationship and state the unit rate.

a.



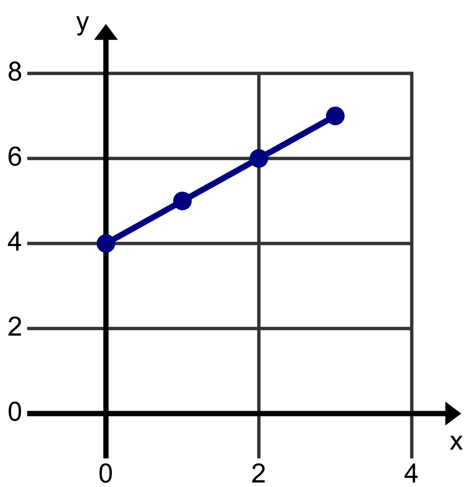
Unit Rate?

b.



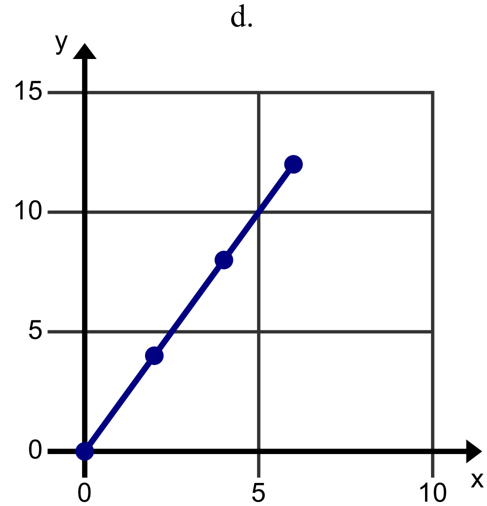
Unit Rate?

c.



Unit Rate?

d.



Unit Rate?

16. How do you think the unit rate/rate of change is related to the steepness of the line? What makes the line steeper? Less steep?

The unit rate describes the steepness of the line. Another word for this is the **slope** of the line. The slope, just like the unit rate, represents the increment change that is occurring on the line. Slope will be investigated in the section 2.3.

2.1c Homework: Proportional Constant, Unit Rate, and Slope

1. The graph below shows the distance a mouse is from her cheese over time. Which sentence is a good match for the graph?

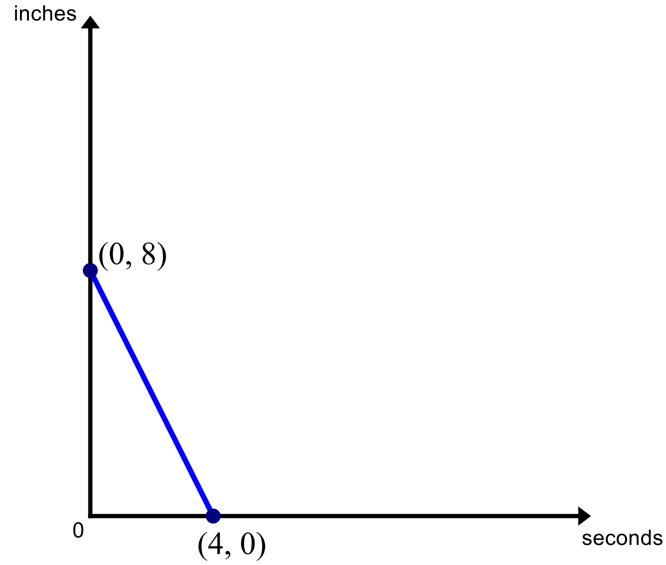
A. The mouse is 8 inches away from the cheese, she sits there and does not move.

B. The mouse is 8 inches away from the cheese, she scurries towards it and reaches it after 4 seconds.

C. The mouse scurries away from the piece of cheese at a rate of 2 inches per second.

D. The mouse scurries away from the piece of cheese at a rate of 4 inches per second.

E. The mouse is 8 inches away from the piece of cheese and scurries away from it a rate of 2 inches per second.



2. Write everything you can say about the mouse and the distance she is from the cheese during this time.

3. Create a table at the right which also tells the story of the graph and your writing.

4. Is this a proportional relationship? If so what is the proportional constant?

5. Find the unit rate in inches per second for this story

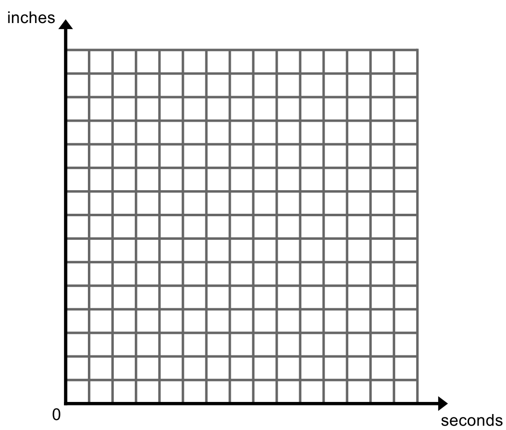
6. Why is there a unit rate for this story but not a proportional constant?

> Recall that even if a straight line is not proportional is still has a unit rate or slope. That means that the change in the variables between two points is in proportion.

7. Sketch a graph for the four stories from number 1 above which you didn't choose in the space provided. Label the graphs by letter to match the story.

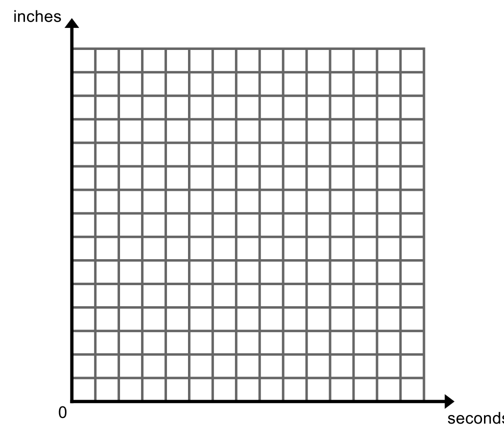
a. Determine if these stories are proportional or not, if they are find the proportional constant.

b. Find the unit rate or rate of change for each story.



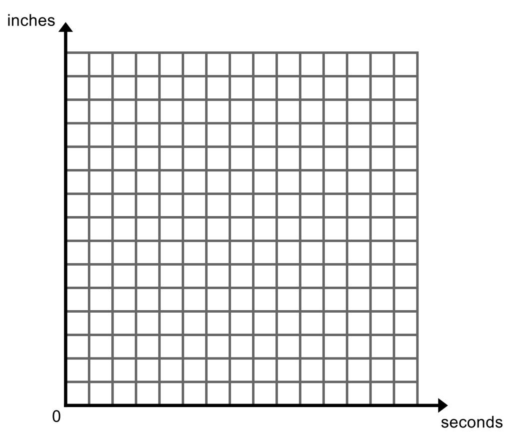
a. Proportional(yes or no):

b. Unit Rate:



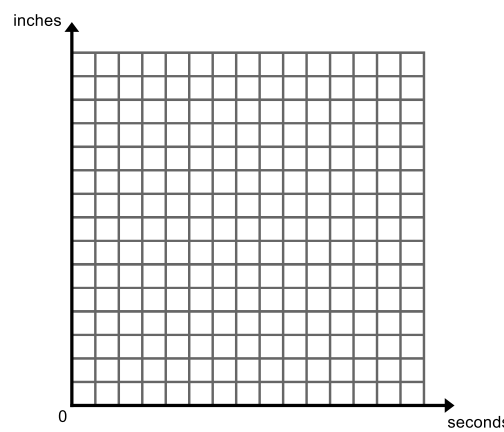
a. Proportional(yes or no):

b. Unit Rate:



a. Proportional(yes or no):

b. Unit Rate:



a. Proportional(yes or no):

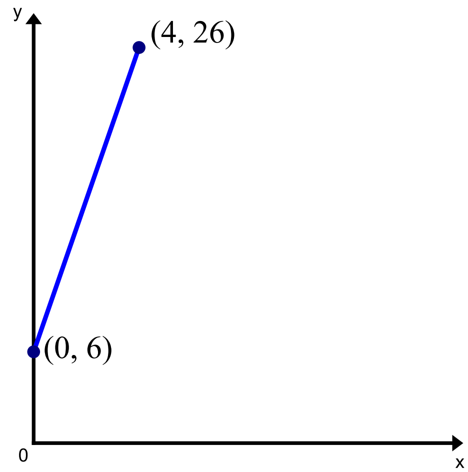
b. Unit Rate:

8. Compare the unit rates with the steepness of each line above, how does the unit rate relate to the steepness or the slope of the line?

10. Use the graph at the right to complete the following.

a. Create a table for this graph,

Time	Distance



b. Write a story for this graph.

c. Which equation matches your story, the graph and the table?

a) $y = 5 + 6x$ b) $y = 4x + 26$

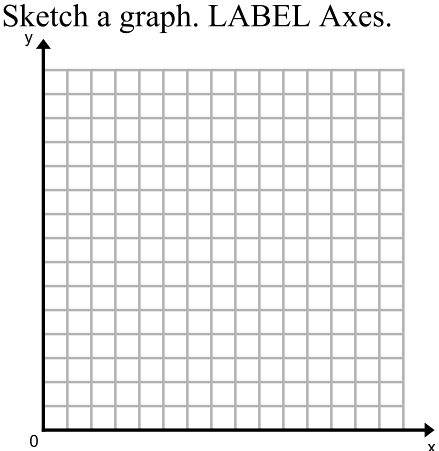
c) $y = 5x + 6$ d) $y = 6x + 4$

For each situation below, fill in tables and graphs and answer questions.

Chocolate cinnamon bears cost $3.00 for 2 pounds.

x ________	*y*_______

Sketch a graph. LABEL Axes.



Is this a proportional relationship?

What is the unit rate:

Equation:

12. Jim had 4 pounds of aluminum cans that he previously collected. He then collected 5 pounds of aluminum cans (to take to the recycling center) in 3 hours.

What is his rate for can collection?

Is this a unit rate also a proportional constant? How do you know?

What will the graph look like using this rate? (sketch if desired)

13. Charlie swam part of a mini-triathlon ½ mile in 1/3 hour.

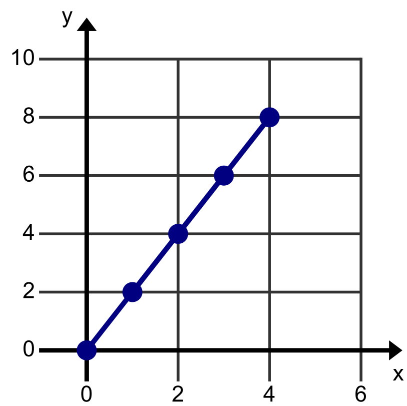
What was his average speed? How do you know?

Is the unit rate a proportional constant? Explain why or why not.

What might the graph look like?
(sketch if desired)

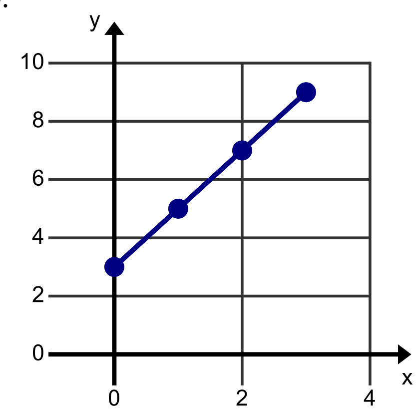
14. Examine the graphs below. If desired, give them a context to help you think (try miles per hour). Determine if the graphs show a proportional relationship and state the unit rate.

a.



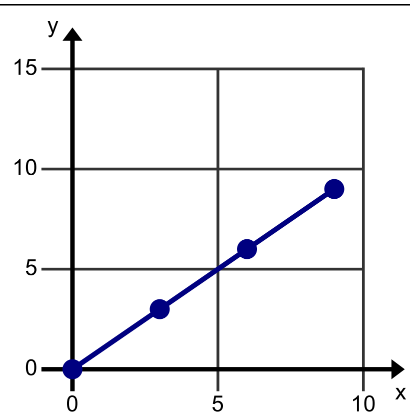
Unit Rate?

b.



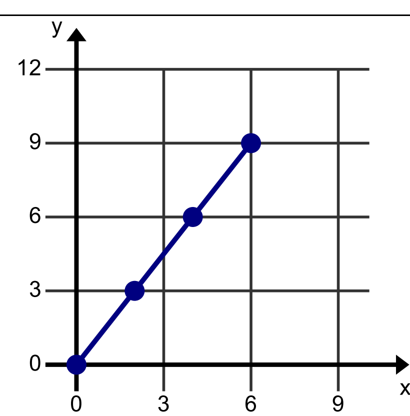
Unit Rate?

c.



Unit Rate?

d.



Unit Rate?

How do you see unit rate in the graph? How is it related to the slope of the line?

2.1d Class Activity: Comparing Proportional Relationships

For the recreational activities below, we will compare the cost per hour by looking at graphs and equations

- Fill in the missing representations, i.e. if the information is given in a table, fill in the story and equation, if the information is given in an equation, fill in the story and table, etc.
- Graph all situations on the given graph form. Label the axes. Label the lines with the situation names.

a. Long Distance Phone Call:

Hours	Cost

Equation: $y=10x$

b. Movies:

Hours	Cost
2	$5

Equation:

c. Concert:

The concert ticket was $75. The concert lasted 3 hours.

Hours	Cost

Equation:

d. Camping

Hours	Cost
24	$6

Equation:

e. Bungee Jumping

It costs $20 to bungee jump for 15 minutes.

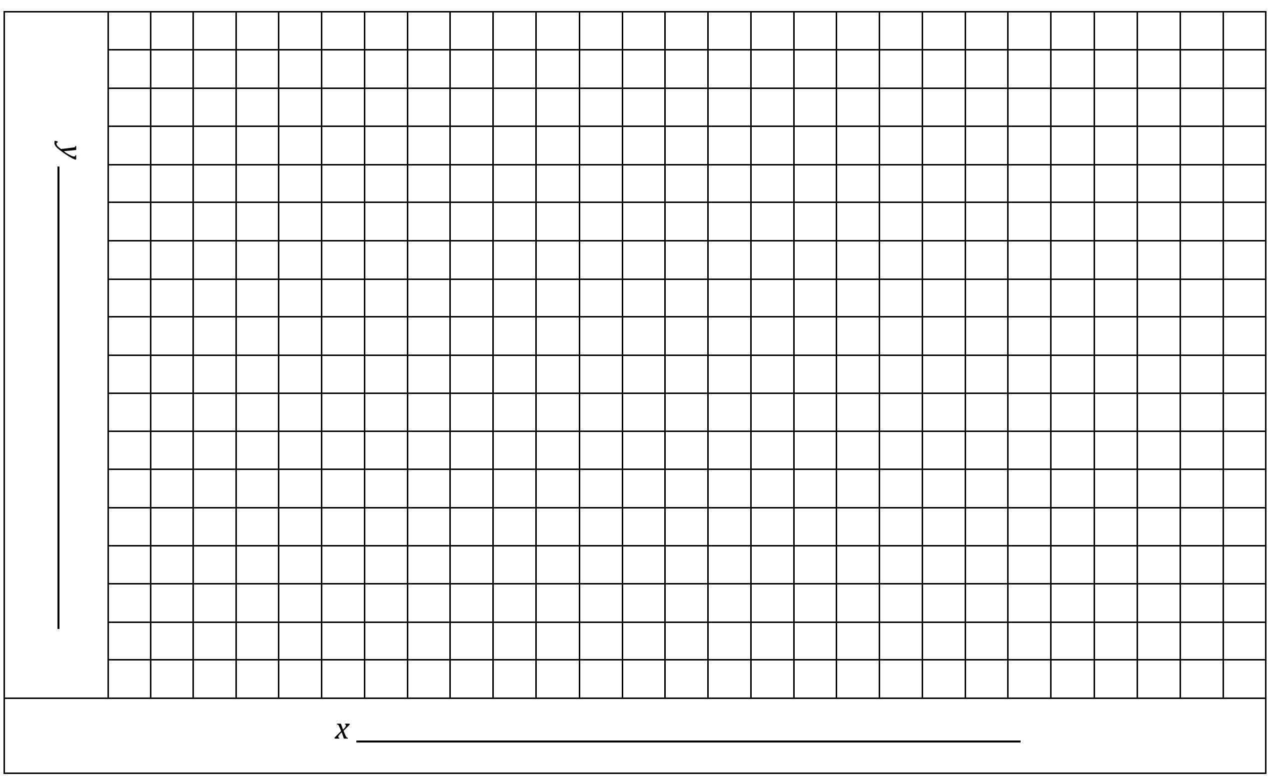
Hours	Cost

Equation:

f.

Hours	Cost

Equation:



2.1d Homework: Comparing Proportional Relationships

1. Order the five activities from highest cost to lowest cost per hour.

1. How do you compare the cost per hour by looking at the graph?

2. How do you compare the cost per hour by looking at the equations?

3. Create a sixth activity in column f. Think of a situation which would be less expensive than Bungee Jumping, but more expensive than the others. Fill in the table and make the graph

Answer the questions below:

4. As the rate gets _______________, the line gets _________________.

5. Camping for 3 hours costs _____.

6. Talking on the phone for 2.5 hours costs _______.

7. Bungee jumping for _______ hours costs $40.

8. For $10 you can do each activity for approximately how much time?

a. Long Distance	
b. Movies	
c. Concert	
d. Camping	
e. Bungee Jumping	
f.	

10. Did you use the tables, equations or graphs to answer questions 5-10? Why?

2.1e Self-Assessment: Section 2.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Deep Understanding, Skill Mastery
1. Find a proportional constant from a table of values, a story or context, and/or a graph			
2. Graph and write equations for a proportional relationship given a table, equation, or contextual situation.			
3. Know that a proportional relationship is a linear graph that goes through the origin.			
4. Understand that the proportional constant or unit rate is the ratio that relates the change in y values to the change in x values, it is the same thing as the slope of a line.			
5. Compare proportional relationships represented in different ways.			

Section 2.2: Investigate The Slope of a Line

Section Overview:

This section uses proportionality to launch an intensive investigation of slope. Once again transformations are integrated, this time through dilations. Students dilate triangle on lines to show that a dilation produces similar triangles and thus the slope is the same between any two distinct points on a non-vertical line. They further their investigation of slope and proportional relationships and derive the slope formula. Adequate time and practice is given for students to solidify their understanding of this crucial aspect of linear relationships.

Concepts and Skills to Master:

By the end of this section, students should be able to:

1. Show that the slope of a line can be calculated as rise/run for any two points on a line.
2. Explain why the slope is the same between any two distinct points on a non-vertical line.
3. Find the slope of a line from a graph
4. Find the slope of a line from a set of points
5. Find the slope of a line from a table
6. Given a context, find slope from various starting points (2 points, table, line, equation).

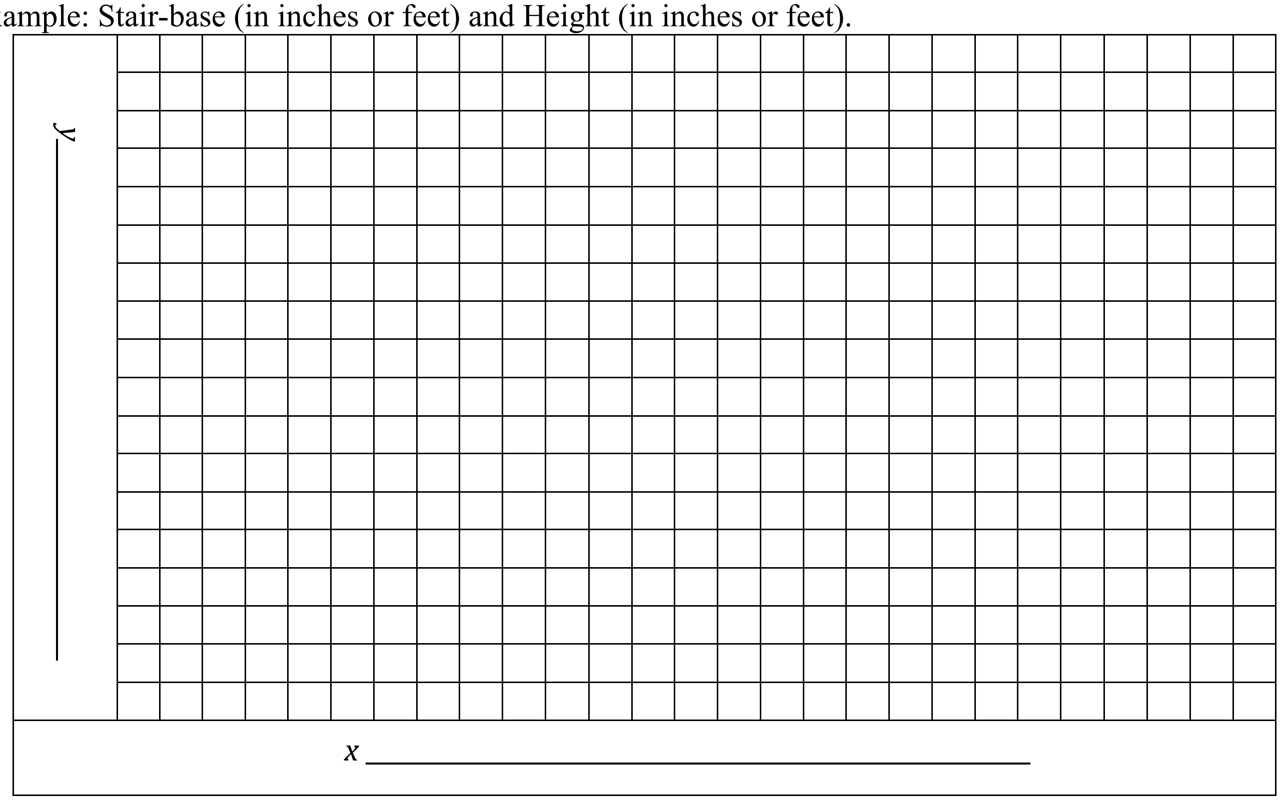
2.2a Class Activity: Building Stairs and Ramps.

Recall from section 2.1 that the unit rate is the same as the slope of a line. The slope of a line describes how steep it is. The following investigation will examine how slope is measured.

On properly built staircases all of the stairs have the same measurements. The important measurements on a stair are what we call the "rise" and the "run". When building a staircase these measurements are chosen carefully to prevent the stairs from being too steep, and to get you to where you need to go.

1. Your group task is to build a set of stairs and a wheelchair ramp at the side. Both the stairs and the ramp will begin at the same place (at ground level) and end at the height of 3 feet. Answer the following questions as you develop your design.
   - How many steps do you want or need?
   - How deep should each step be (we'll call this the run)? Why do you want this run depth?
   - How tall will each step be (we'll call this the rise)? Why do you want this rise height?
   - What is the total distance (total depth for all steps) you will need (at the base) for all of the stairs—this would be a measurement at ground level from stair/ramp start point to stair/ramp end point?

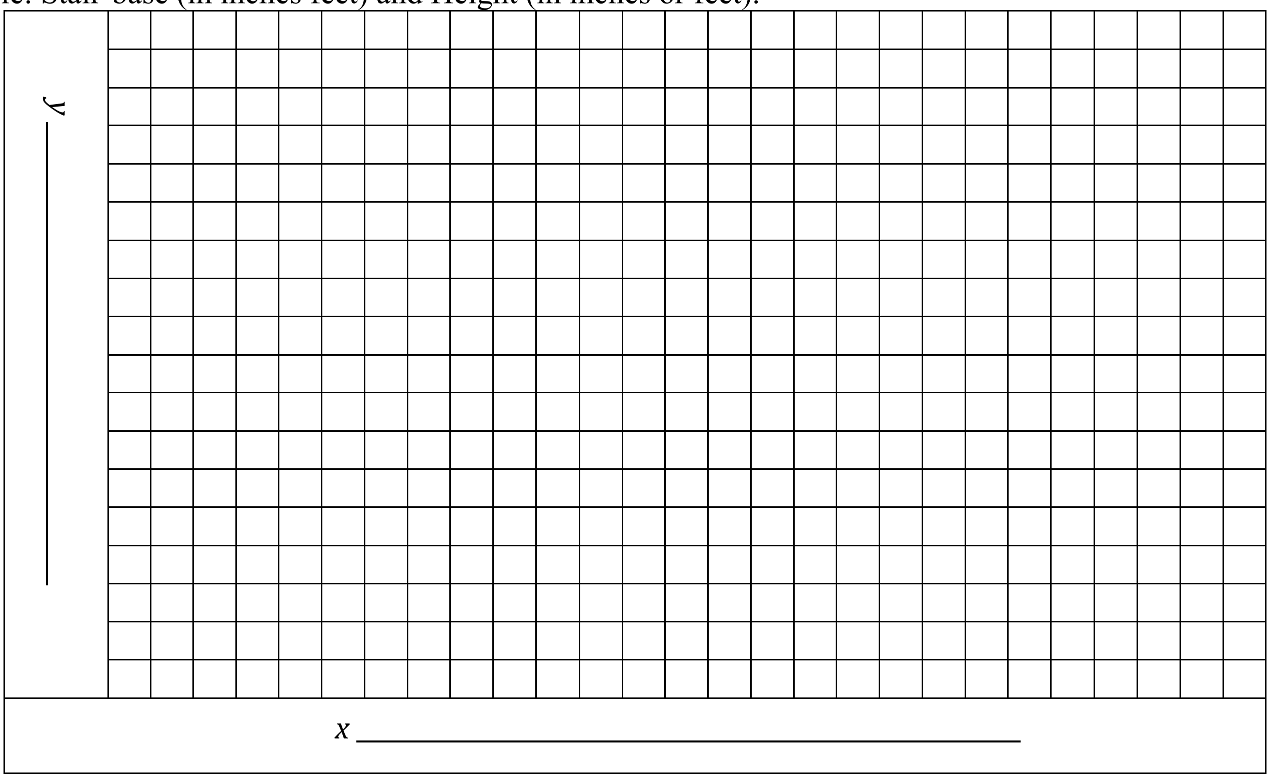
2. Sketch the ramp (as viewed from the side) on graph paper below. Label and sketch the base and height, for example: Stair-base (in inches or feet) and Height (in inches or feet).



3. From the sketch , find and record the following measurements.

	Rise height (total height you've climbed on the ramp at this point)	Run depth (total distance – at ground level – covered from stair-base beginning)	Ratio $\frac{rise}{run}$
2 feet in from the start of the ramp			
3 feet in from the start of the ramp			
Where the ramp meets the top			

4. Sketch the stairs (as viewed from the side) on graph paper below. Label the sketch base and height, for example: Stair-base (in inches feet) and Height (in inches or feet).



5. From the sketch above, find the following measurements. Record.

	Rise height (total height you've climbed at this point)	Run depth (total distance covered from stair-base beginning)	Ratio $\frac{rise}{run}$	Reduced ratio
At the first step				
At the third step				
At the last step				

6. What do you notice about the Ratio column for both the ramp and the stairs?

7. On your stair drawing, draw a line from the origin (0,0) through the very tip of each stair. Now look at your ramp drawing. What do you observe?

2.2a Homework: Measuring the Slope of Stairs and Ramps

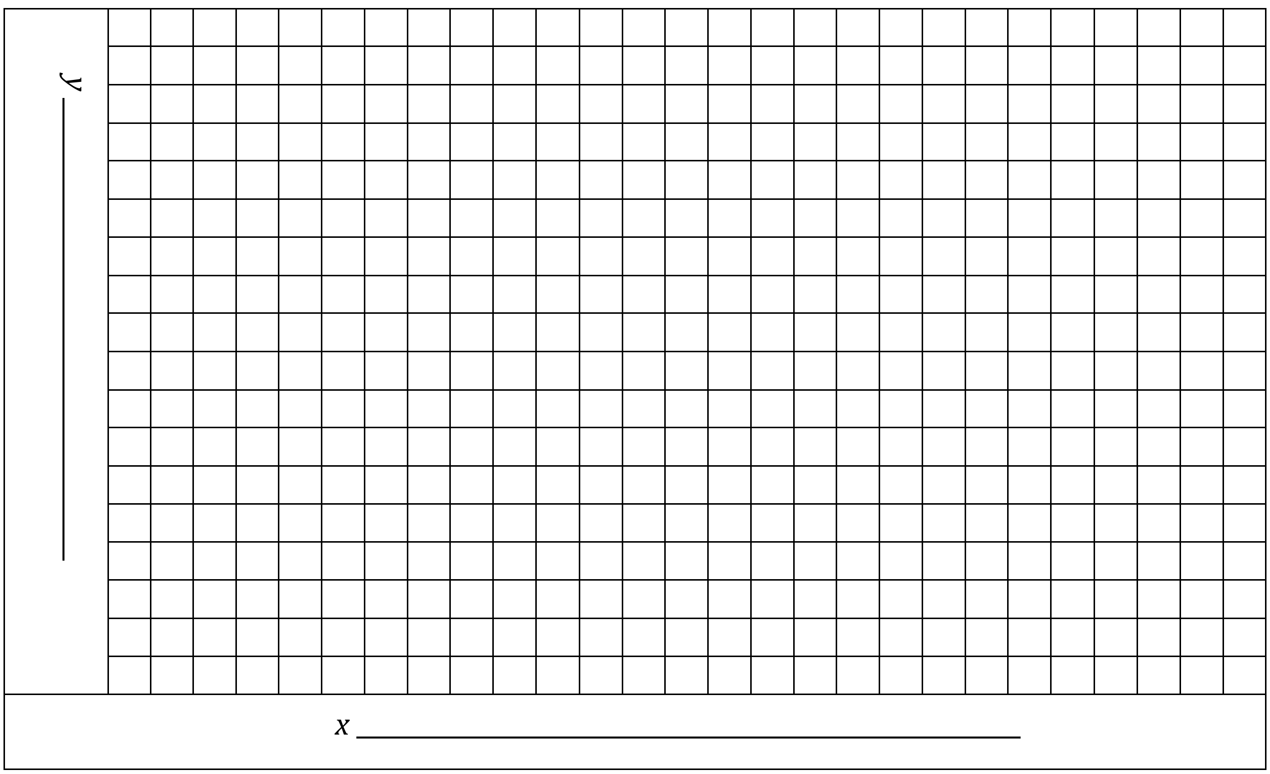
Just like staircases, the measurement of the steepness of a line is also very important information. For each step you climb, you move up ***y*** inches and forward ***x*** inches. We can find the slope of each line representing a staircase using the ratio: ***rise/run*** or ***y/x***, and then simplifying this fraction. You are going to further investigate slope in the homework activity below.

Preparations:

- Decide on group members.
- Decide what set of stairs each of you will measure.
- If you don't have a ruler at home, then create one before you leave the classroom—just measure off inches on a piece of paper.

2. Measure the rise and the run for one stair step (in inches). Record here. Rise________ Run________

3. Draw the stairs (side view) on the graph paper below. Label the axes.



4. What is the $\dfrac{rise(inches)}{run(inches)}$ for your stairs?

5. Draw a connecting line from the origin (0,0) and through the tip of each stair step. This line shows the slope of your stairs. What is the slope of your stairs $\dfrac{rise(inches)}{run(inches)}$?

6. Explain what would happen to the slope of the line for your stairs if the rise of your stairs was higher or lower?

7. Why do you think that slope has been defined as $\dfrac{rise}{run}$ instead of the opposite?

8. Create a table to show height as related to number of stair steps for your stairs.

Total Height (y)									
Steps (x)	1	2	3	4	5	6	7	8	9

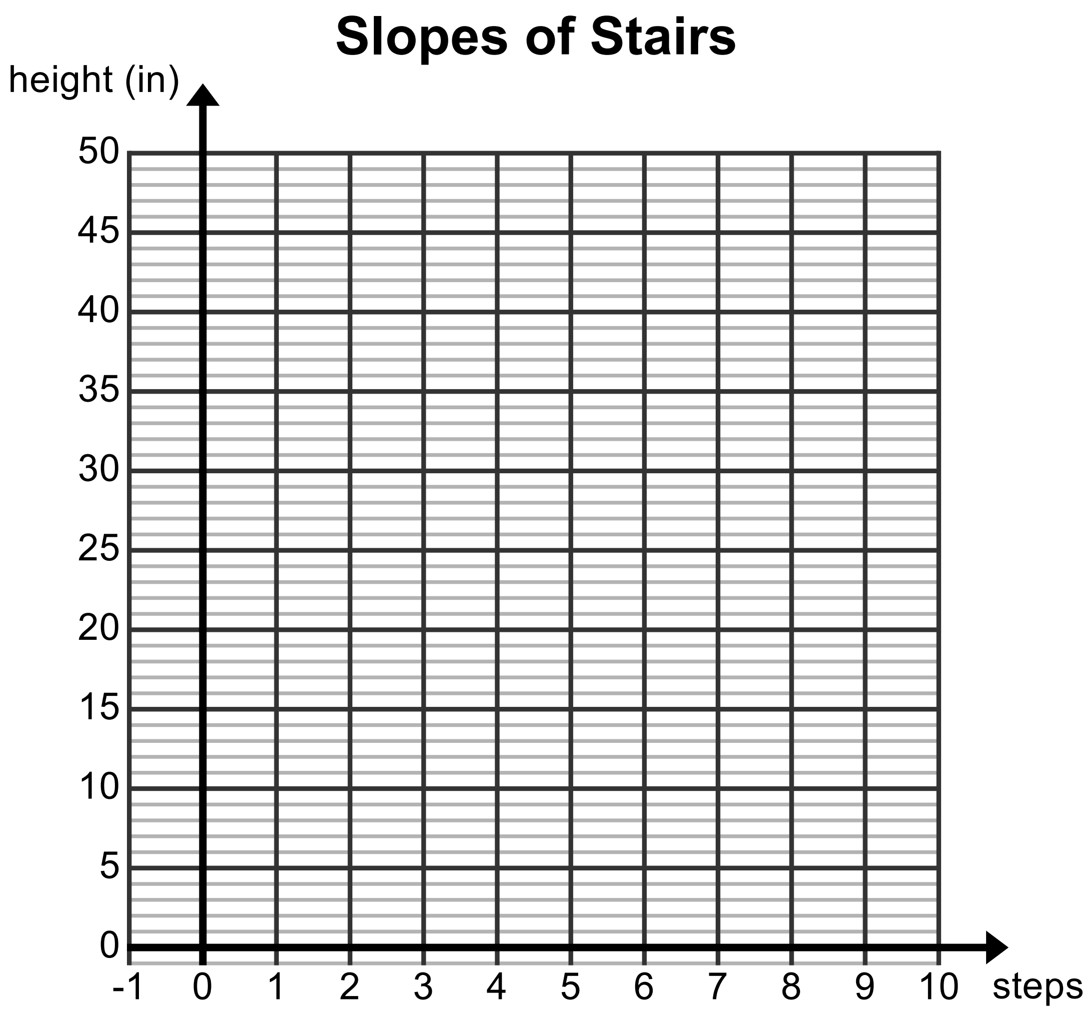
9. Now what is your slope? (consider 1 as the run).

10. Write the equation for finding the height of your stairs at any point.

Height= _____________ OR $y =$ _________

11. How high would you be if you climbed 50 of your steps?

12. Now create the graph of your stairs based on the *run* as the number of steps instead of the *run* in inches.



To be completed during homework debriefing.

13. Record the slopes from your group members (from #8 above).

a. Name_____________ Slope ___________

b. Name_____________ Slope ___________

c. Name_____________ Slope ___________

d. Name_____________ Slope ___________

14. Using the slopes given to you, draw the lines of those slopes on your graph above (#12 above).
    - Use a different color for each group member.
    - LABEL the lines with the person's name.

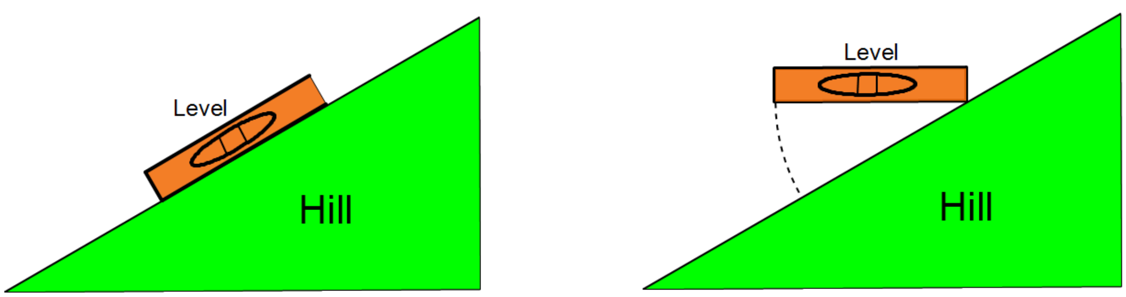
15. What do you notice about the steepness of the lines as related to the slopes?

16. If you didn't have the graph to look at, only the ratios you just calculated, how would you know which staircase would be the steepest?

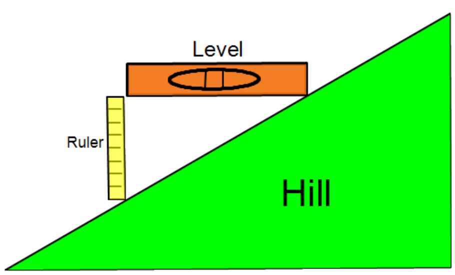
Extra Credit: Measure the slope of a hill

Instructions for measuring the grade of a hill or a road:

1. Rest the level (or straight edge) on the ground. Lift up the lower end of the level/straight edge (i.e, the end nearest the bottom of the hill) until the level measures level.



2. While holding the level/straight edge still in this position, measure the distance between the end of the level and the ground using the ruler, as shown in the image below.



3. Calculate the grade by dividing the distance measured with the ruler (the "rise") by the length of the level or straight edge (the "run") and multiplying by 100:

2.2b Class work: Dilations and Similar Figures

When an object, such as a line, is moved in space it is called a transformation. A special type of transformation is called a dilation. A dilation transforms an object in space from the center of dilation, usually the origin, by a scale factor called r. The dilation moves every point on the object so that the point is r times away from the origin as it was originally. This means that the object is enlarged or shrunk.

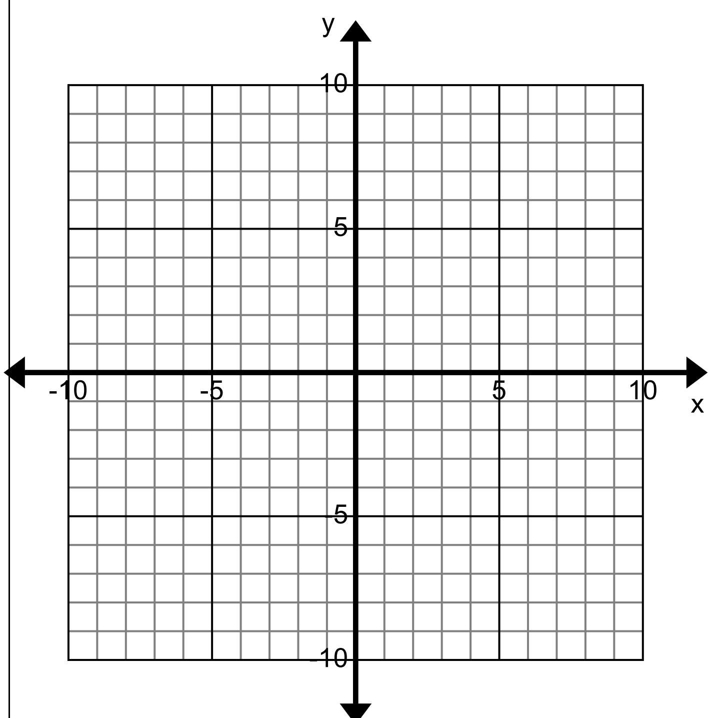
For example, if you dilate the set of points (0,0), (0,4),(3, 0), and (3,4) with a scale factor of 2 and the center is at the origin (0,0) the distance of each point from the center will be 2 times as long as it was originally. Algebraically this means that you multiply each point by 2.

$$(x, y) \to (2x, 2y)$$

To confirm this, investigate this transformation below.

1.

a. Graph and connect the ordered pairs (0,0), (0,4), (3, 0), and (3,4)



c. Find the length of each segment and dilate it by a scale factor of two. This means to multiply each length by two. Draw the new lengths from the center of dilation (the origin) in a different color.

The original object is called the ***pre-image*** and the transformed object is called the ***image***.

d. Compare the size of the pre-image with the image.

e. Dilate by a scale factor of 2 algebraically using the ordered pairs. That it is $(x, y) \to (2x, 2y)$. Write the ordered pairs below.

e. Graph your new ordered pairs for the image.

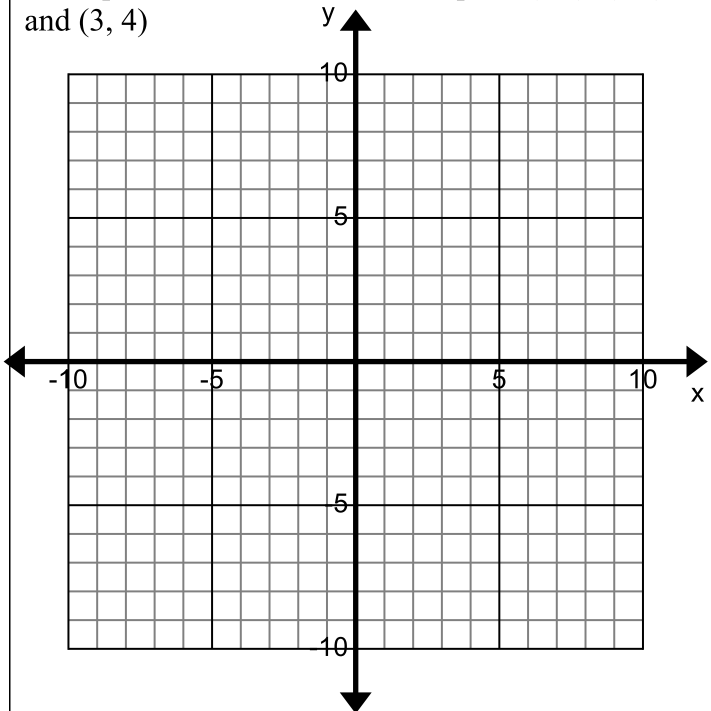
f. What do you observe about the transformation when you do it graphically and algebraically?

g. How do the lines that correspond to one another in the image and pre-image compare?

A special notation is used to differentiate between the pre-image and the image. If the pre-image is called ***A*** then the image is called ***A'***, pronounced "A prime".

2. Try another shape to see what kind of relationship exists between the pre-image and the image.

a. Graph and connect the ordered pairs (0,0), (3,0), and (3, 4)



c. Find the length of each segment and dilate it by a scale factor of $\frac{1}{2}$. (The length of the hypotenuse is 5) This means to multiply each length by $\frac{1}{2}$. Draw the new lengths from the center of dilation (the origin) in a different color.

d. Labe the pre-image A and the image A'. Compare the size of the pre-image with the image.

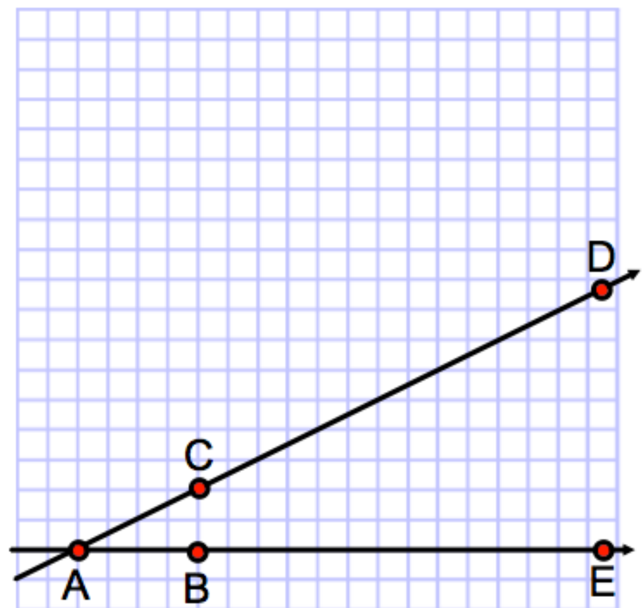
e. Dilate by a scale factor of $\frac{1}{2}$ algebraically using the ordered pairs. That it is $(x, y) \rightarrow (2x, 2y)$. Write the ordered pairs below.

e. Graph your new ordered pairs for the image.

f. What do you observe about the transformation when you do it graphically and algebraically?

g. How do the lines that correspond to one another in the image and pre-image compare?

Now that we know how a dilation works investigate what kind of relationship is formed between the pre-image and the image after the dilation.

3) Using the figure below.	
a. Connect point B to C b. Double the length of $\overline{AB}$ on the line $\overleftrightarrow{AE}$. Label the new segment $\overline{AB'}$ c. Double the length of $\overline{AC}$ on the line $\overleftrightarrow{AD}$. Label the new segment $\overline{AC'}$. d. Connect B' to C'.	

4. What do you notice about $\overline{B'C'}$ in relationship to $\overline{BC}$?

5. What do you notice about the size of the two triangles?

6. Write a ratio that compares the corresponding parts of the pre-image with the image.

7. What kind of figures are these triangles in relationship to one another? Explain how you know.

2.2b Homework: Dilations and Similar Figures

Continue your investigation of dilations and the relationships between the pre-image and image below.

1. Using the figure below:

a. Connect point B to C

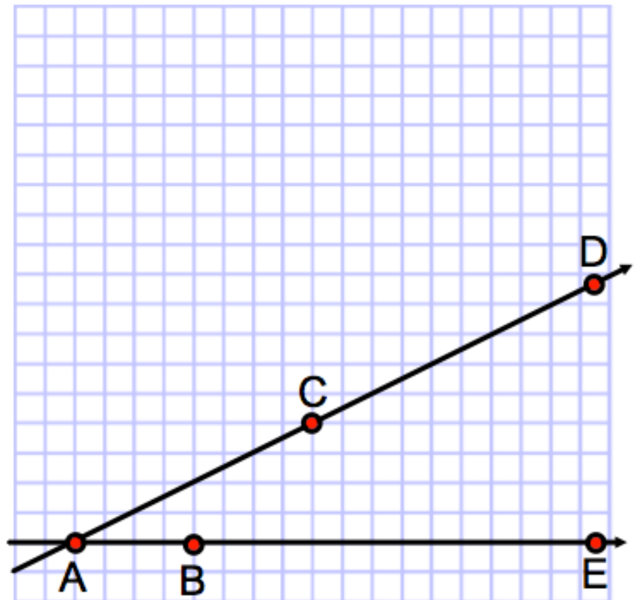
b. Double the length of $\overline{AB}$ on the line $\overleftrightarrow{AE}$. Label the new segment $\overline{AB'}$

c. Double the length of $\overline{AC}$ on the line $\overleftrightarrow{AD}$. Label the new segment $\overline{AC'}$.

d. Connect B' to C'.

2. What do you notice about $\overline{B'C'}$ in relationship to $\overline{BC}$?

3. What kind of figures are these triangles in relationship to one another? Explain how you know.



4. Using the figure below:

a) Connect point B to C

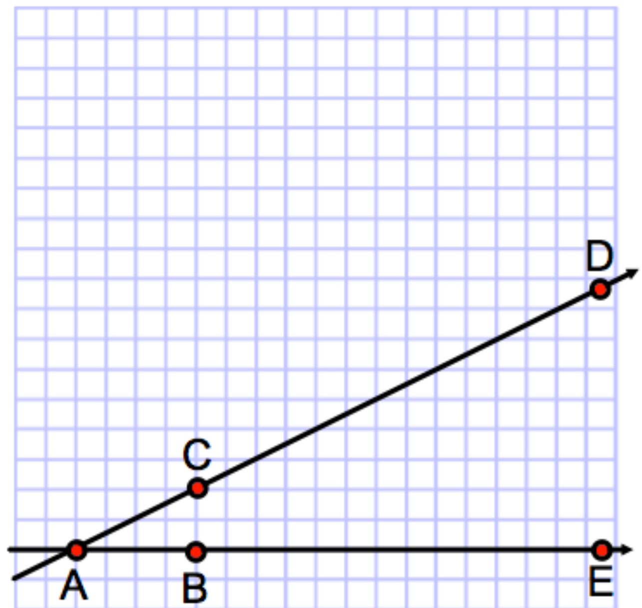
b) HALF the length of $\overline{AB}$ on the line $\overleftrightarrow{AE}$.
Label the new segment $\overline{AB'}$

c) HALF the length of $\overline{AC}$ on the line $\overleftrightarrow{AD}$.
Label the new segment $\overline{AC'}$.

d) Connect B' to C'.

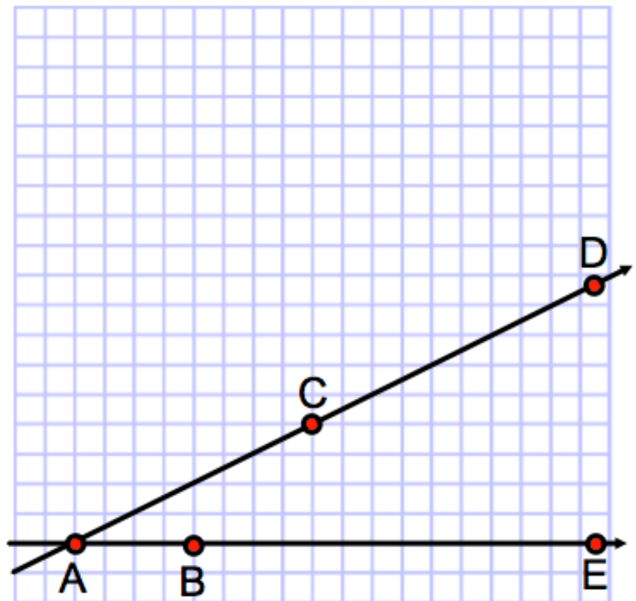
5. What do you notice about $\overline{B'C'}$ in relationship to $\overline{BC}$?

6. What kind of figures are these triangles in relationship to one another? Explain how you know.



7. Using the figure below:

a. Connect point B to C



b. HALF the length of $\overline{AB}$ on the line $\overleftrightarrow{AE}$.
Label the new segment $\overline{AB'}$

c. HALF the length of $\overline{AC}$ on the line $\overleftrightarrow{AD}$.
Label the new segment $\overline{AC'}$.

d. Connect B’ to C’.

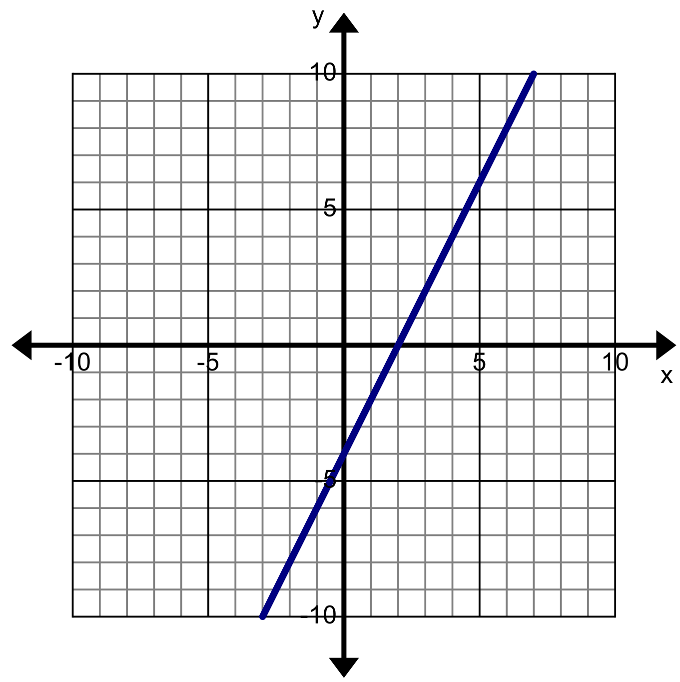
8. What do you notice about $\overline{B'C'}$ in relationship to $\overline{BC}$?

9. What kind of figures are these triangles in relationship to one another? Explain how you know.

A dilation will produce an image that has corresponding parts that are proportional to the pre-image. Thus a dilation produces similar figures. Also the lines that make up the pre-image and the image are parallel and they have the same slope. We will use the information to show that you can calculate slope given any two points on a line in the next section.

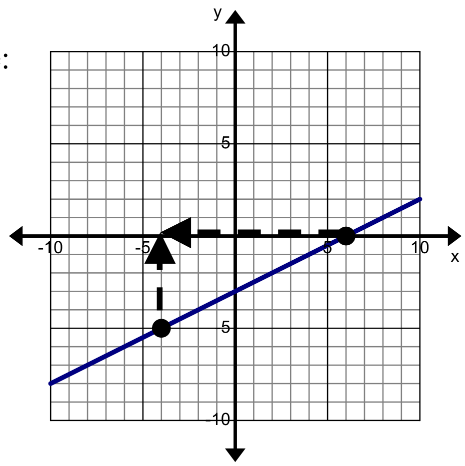
2.2c Classwork: Similar Triangles and Slope

1. On the line to the right choose any two points that fall on the line. (To make your examination easier choose two points that fall on an intersections of the gridlines).



From the two points create a right triangle, the line itself will be the hypotenuse and the legs will extend from the two points and meet at a right angle. An example is shown below.

Example:



2. Compare the points that you choose and your triangle with someone in your class. Discuss the following;
Did you both choose the same points?
How are your triangles the same?
How are your triangles different?
What relationship exists between your triangles?

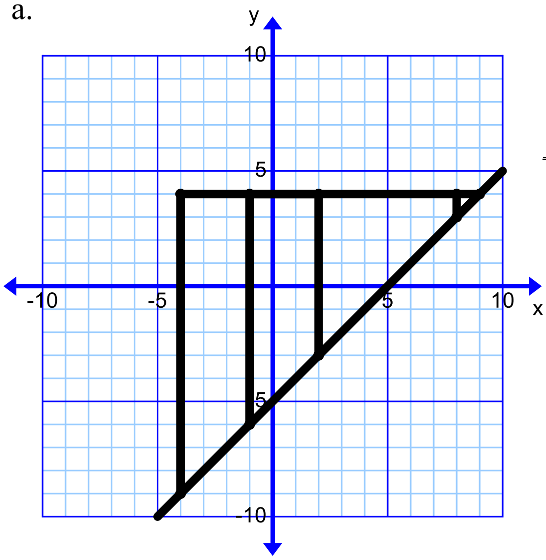
Given any two triangles with hypotenuse on the given line and legs horizontal and vertical, then there is a dilation that takes one on the other. In particular, the lengths of corresponding sides are all multiplied by the factor of the dilation, and so the ratio of the length of the vertical leg to the horizontal leg is the same for both triangles.

The graphs given below have several different triangles formed from two points that lie on the line. These triangles are dilations of one another and are similar. Answer the questions below about these lines to observe what this tells us about slope.

3. In each graph below, how many right triangles do you see?

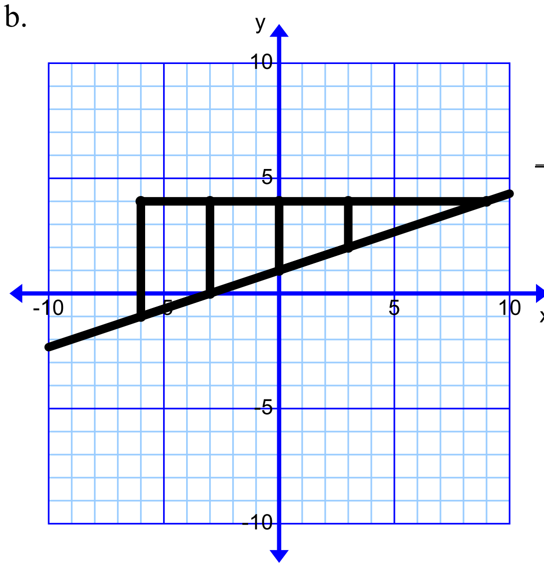
- Trace the triangles by color.
- For each triangle write a ratio comparing the lengths of its legs or $\frac{height}{base}$. Then simplify the ratio $\frac{height}{base} = \underline{\qquad}$.
- In the future we will refer to the ratio as $\frac{rise}{run}$, instead of $\frac{height}{base}$.

a.



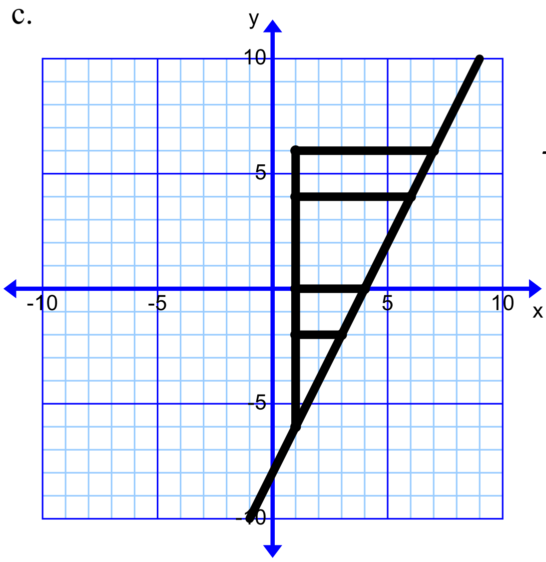
Ratios

b.



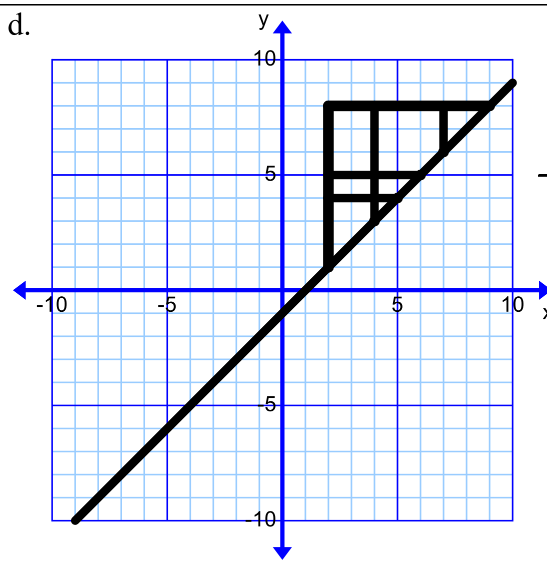
Ratios

c.



Ratios

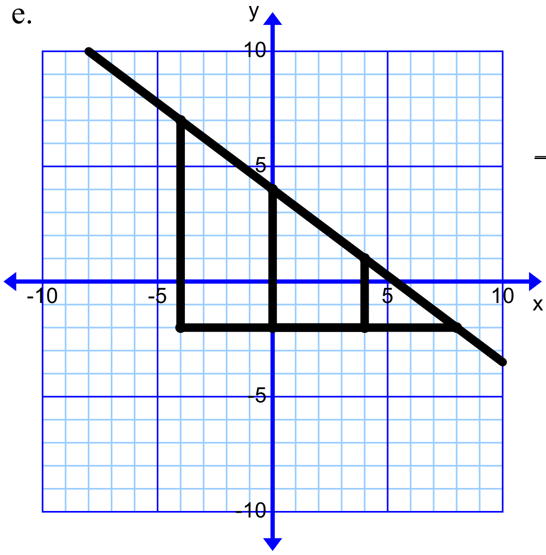
d.



Ratios

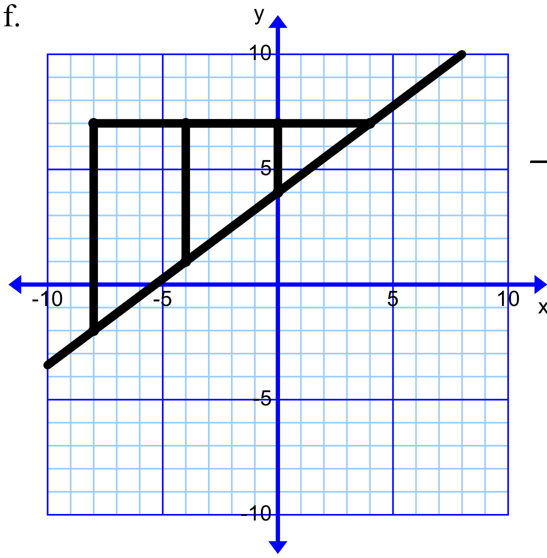
- Do the ratios (rise to run) always simplify to the same fraction (even in the negative quadrant)? Why or why not?
- How does the "*rise over run ratio*" describe the steepness of the line?

e.



Ratios

f.



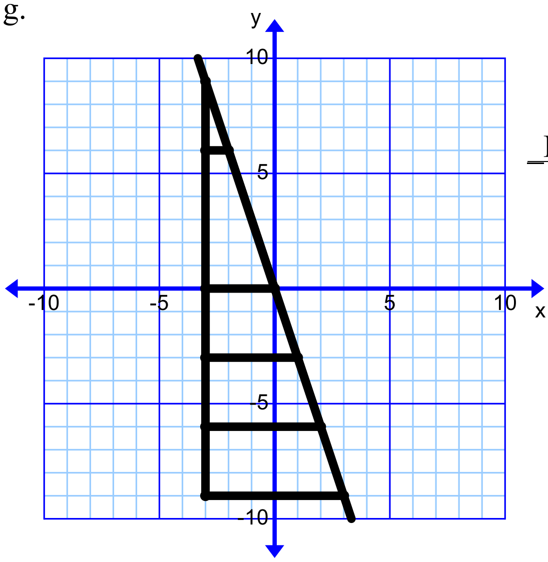
Ratios

- Why do graphs *e* and *f* have the same ratio but they are different lines?

- How could you differentiate between the slopes of these lines?

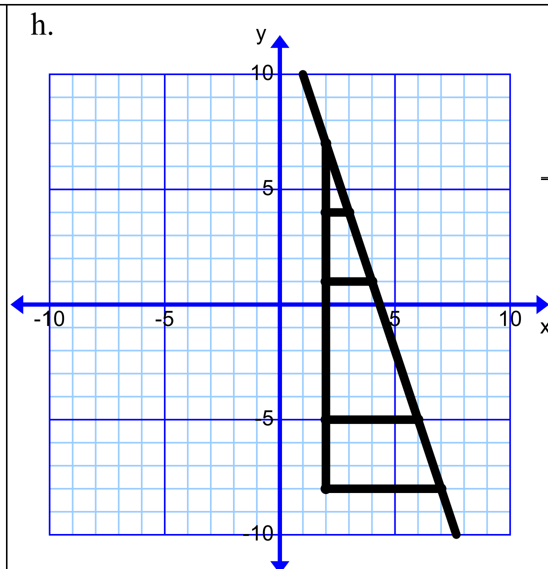
- How does the rise related to the run of a negative slope affect the steepness of the line?

g.



Ratios

h.



Ratios

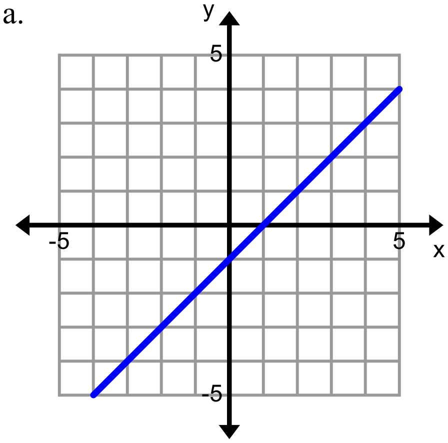
- Are the ratios for graphs *g* and *h* positive or negative? How do you know?

- Why are the slopes for graphs *g* and *h* the same if they are different lines?

2.2c Class Activity: Similar Triangles and Slope

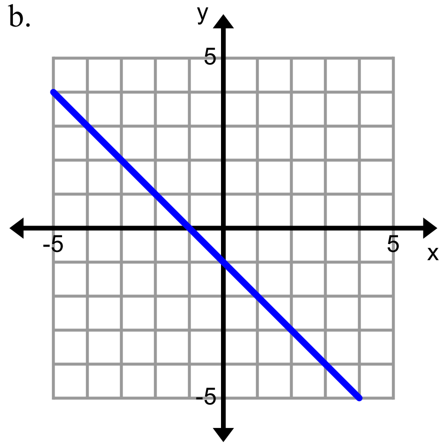
1. For each line graphed below,
   - Draw a Right Triangle to calculate the slope of the line. The slope of a line is denoted by the letter *m*. Thus $slope = m = \frac{rise}{run}$
   - Label each triangle with a ratio and simplify the ratio $m = \frac{rise}{run} = \_\_\_\_$

a.



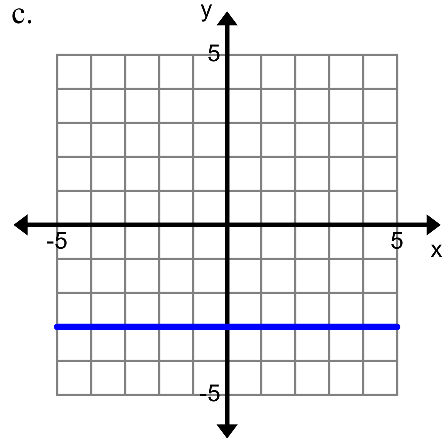
$m = \frac{rise}{run} = \_\_\_\_$

b.



$m = \frac{rise}{run} = \_\_\_\_$

c.

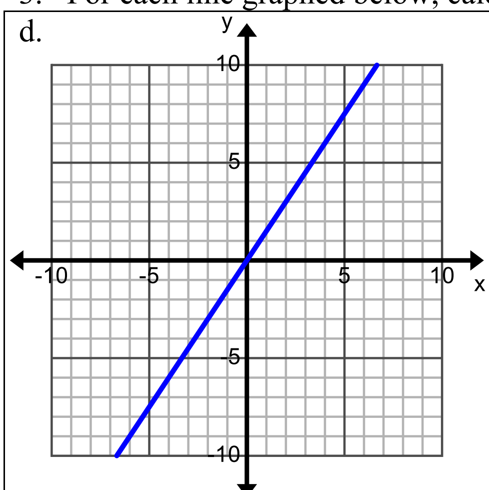


$m = \frac{rise}{run} = \_\_\_\_$

2. What does the sign of the slope tell us about the line?

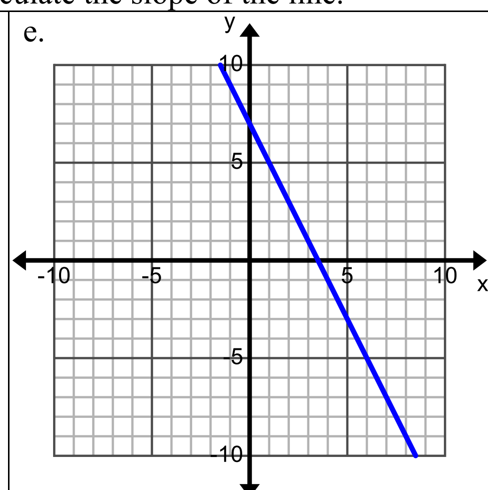
3. For each line graphed below, calculate the slope of the line.

d.



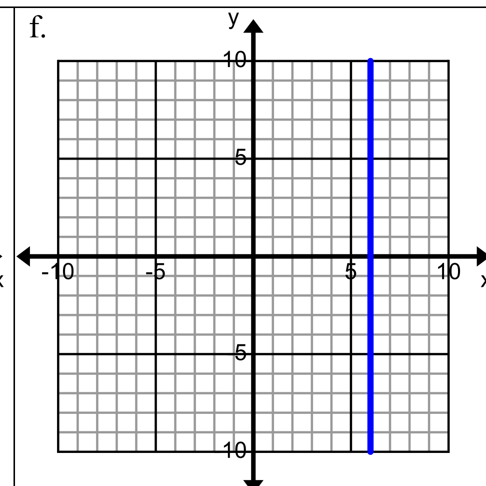
$m = \frac{rise}{run} = \_\_\_\_$

e.



$m = \frac{rise}{run} = \_\_\_\_$

f.

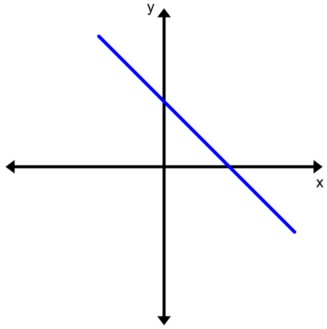
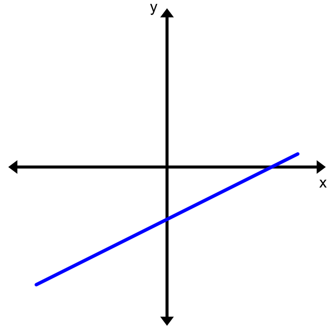
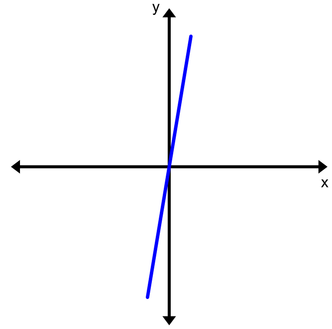
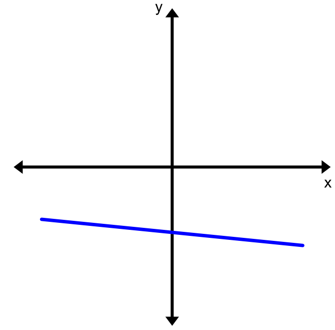


$m = \frac{rise}{run} = \_\_\_\_$

4. Briefly, explain how to calculate slope when looking at a graph.

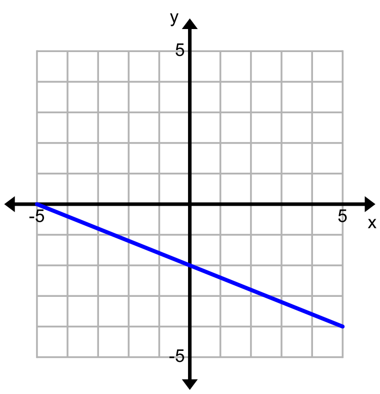
2.2d Class Activity: Finding Slope from Graphs

1. Do the graphs below have positive or negative slopes? How do you know?

a.	b.	c.	d.
			
Positive or negative?			
e. Explain how you know whether a line of a graph has a positive or negative slope.			

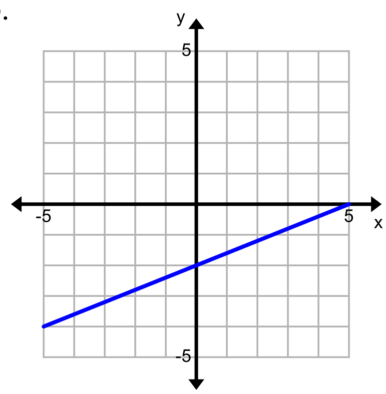
2. For each line graphed below,
   - Draw a right triangle to calculate the slope of the line.
   - Label each triangle with a ratio and simplify the ratio $m = \frac{rise}{run} = \_\_\_\_$

a.



$\frac{rise}{run} = \_\_\_\_$

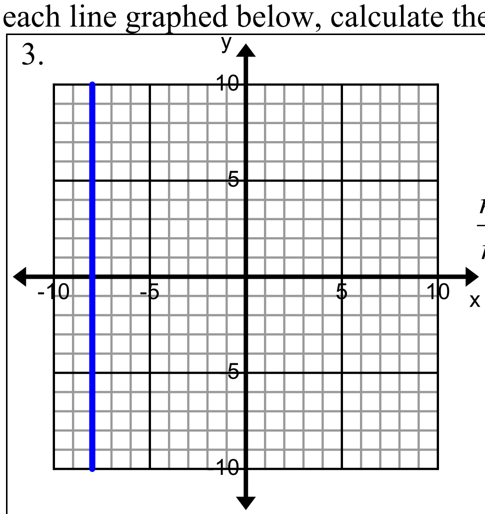
b.



$\frac{rise}{run} = \_\_\_\_$

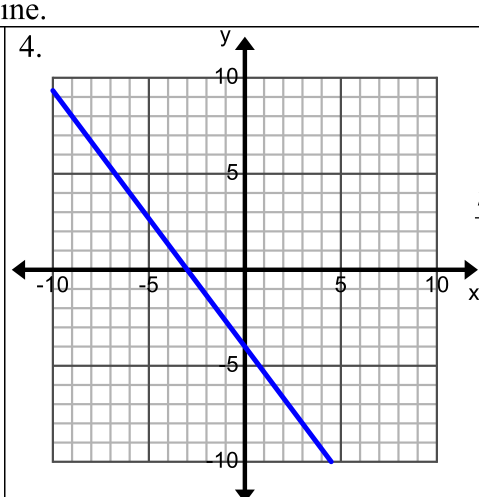
For each line graphed below, calculate the slope of the line.

3.



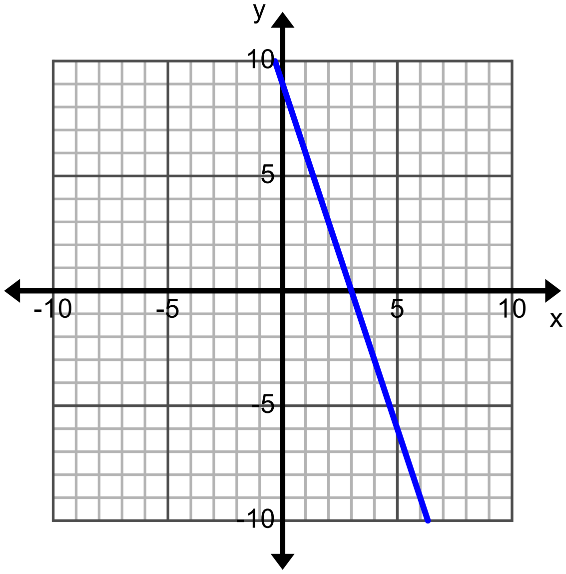
$\frac{rise}{run} = \_\_\_\_$

4.



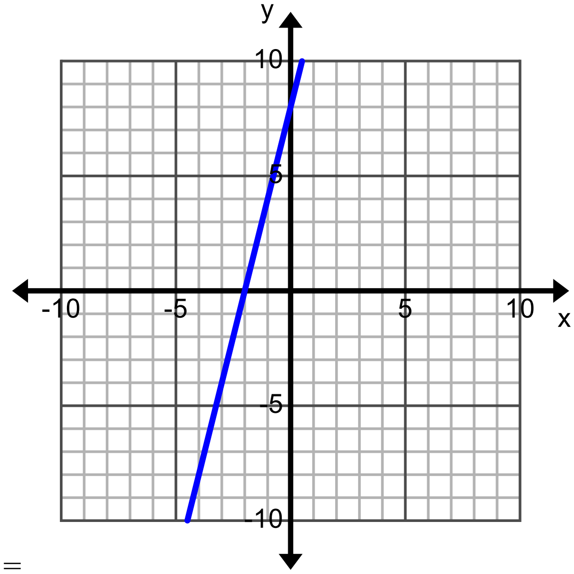
$\frac{rise}{run} = \_\_\_\_$

5.



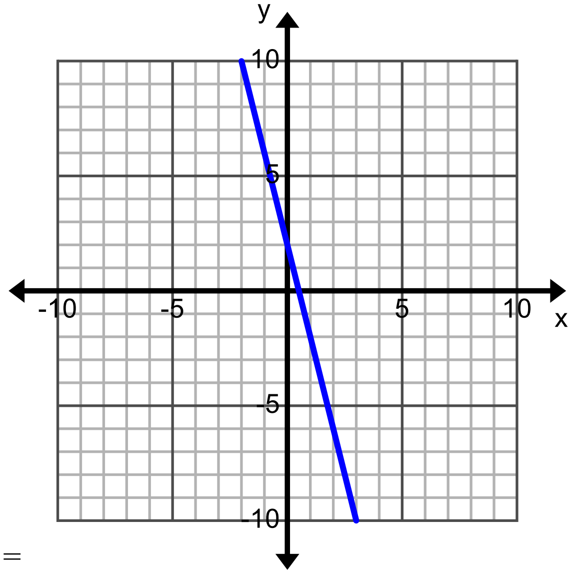
m=

6.



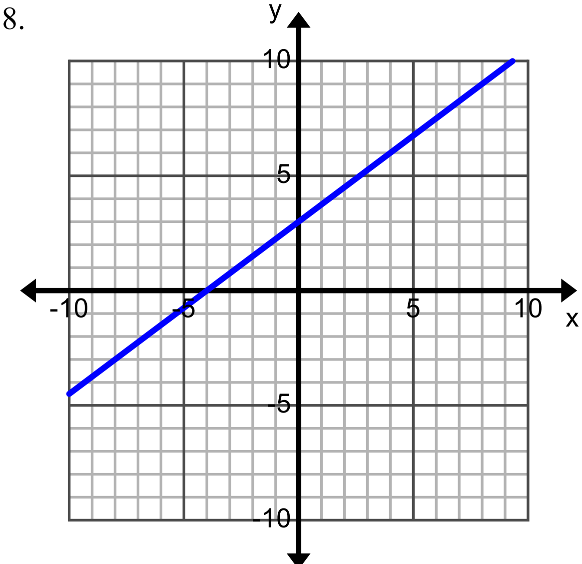
m=

7.



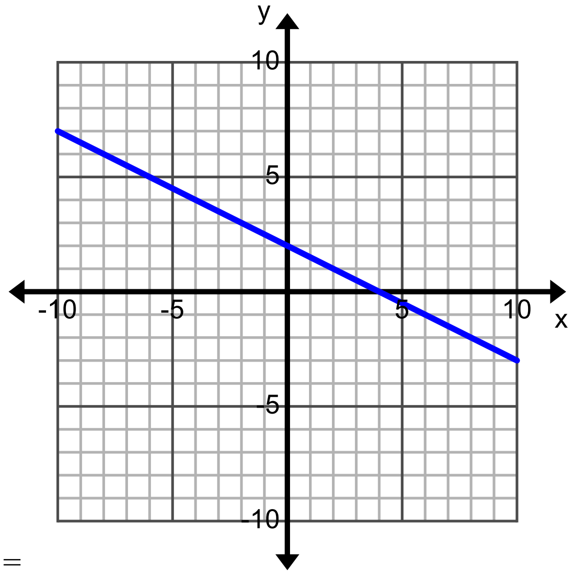
m=

8.



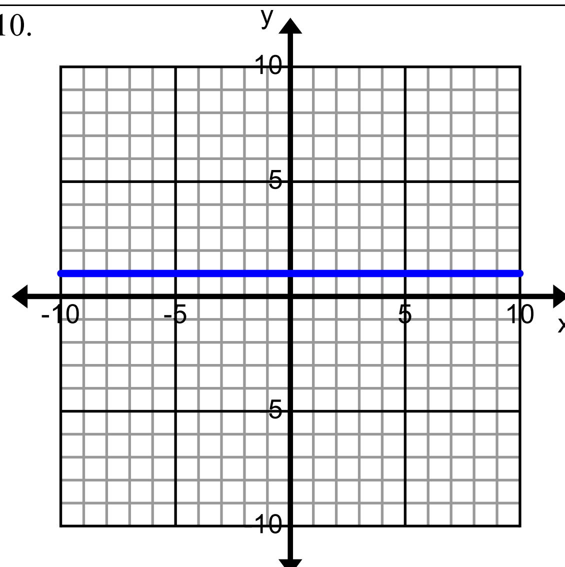
m=

9.



m=

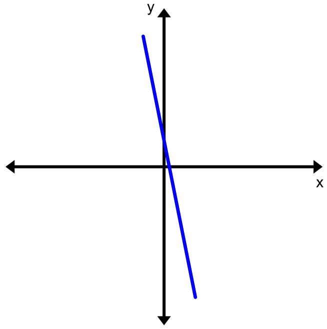
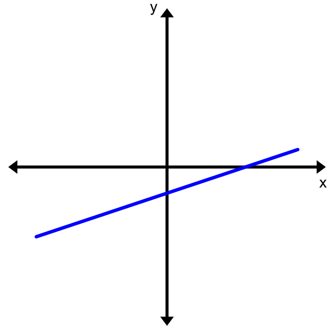
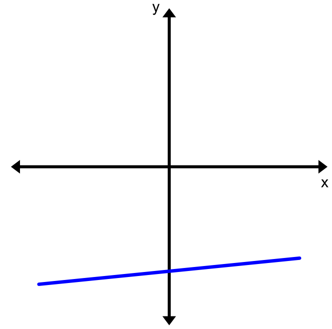
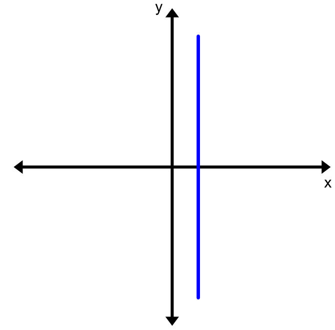
10.



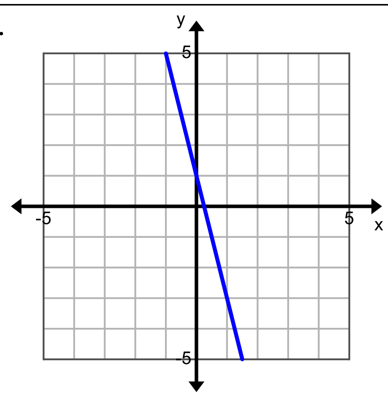
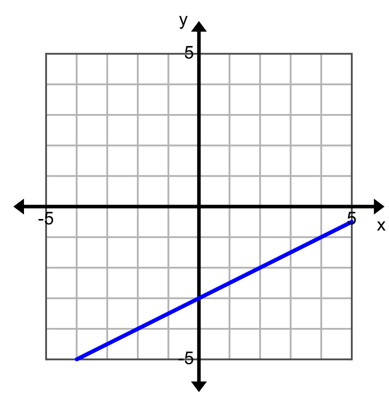
m=

2.2d Homework: Finding Slope from Graphs

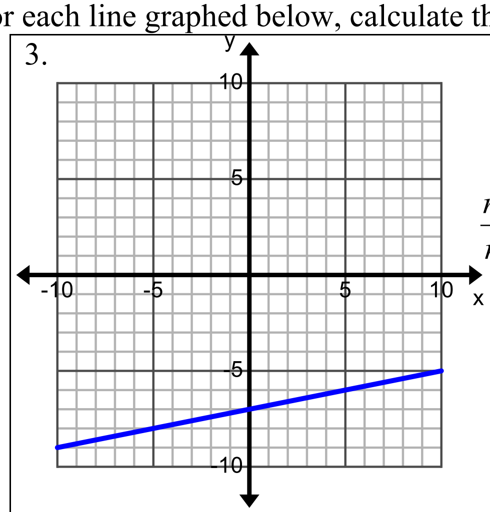
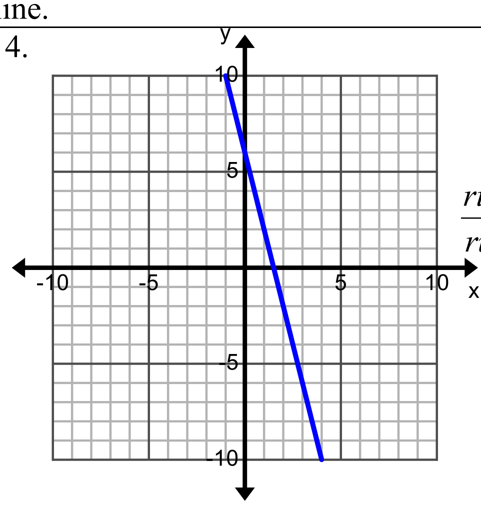
1. Do the graphs below have positive or negative slopes? How do you know?

a.	b.	c.	d.
			
Positive or negative?			
e. Explain how you know whether a line of a graph has a positive or negative slope.			

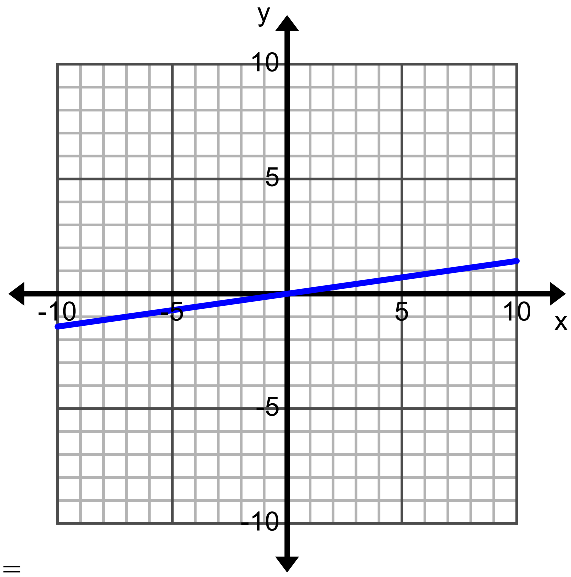
2. For each line graphed below,
   - Draw a right triangle to calculate the slope of the line.
   - Label each triangle with a ratio and simplify the ratio $m = \dfrac{rise}{run} = \_\_\_\_\_$

a.	b.
 $\dfrac{rise}{run} = \_\_\_\_\_$	 $\dfrac{rise}{run} = \_\_\_\_\_$

For each line graphed below, calculate the slope of the line.

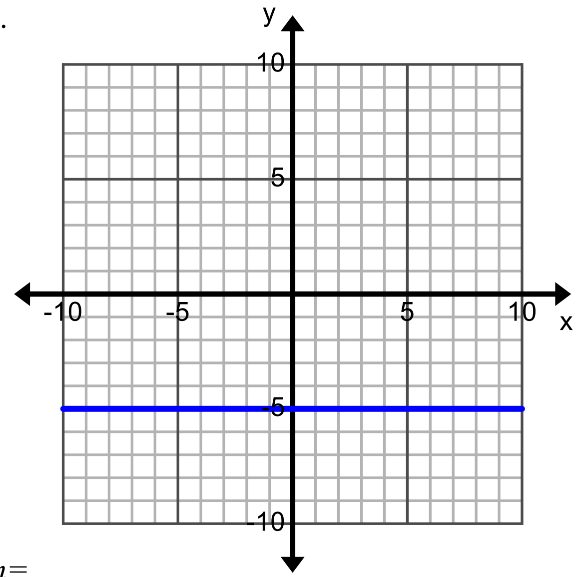
3.	4.
 $\dfrac{rise}{run} = \_\_\_\_\_$	 $\dfrac{rise}{run} = \_\_\_\_\_$

5.



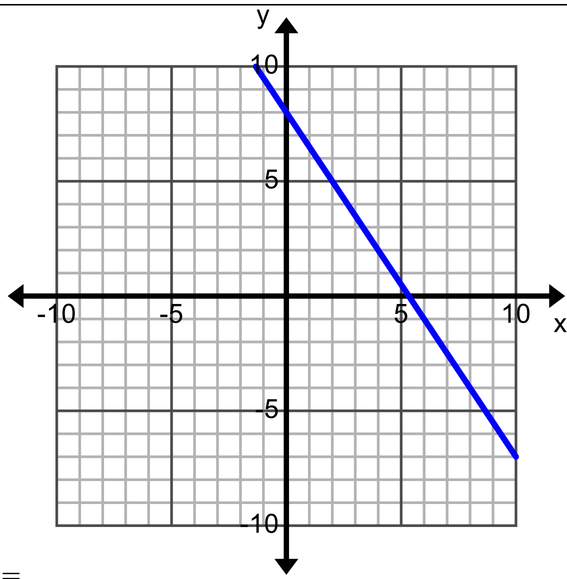
$m=$

6.



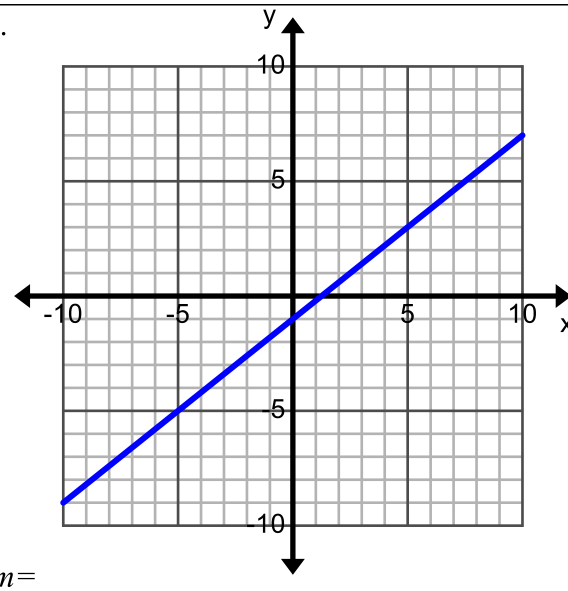
$m=$

7.



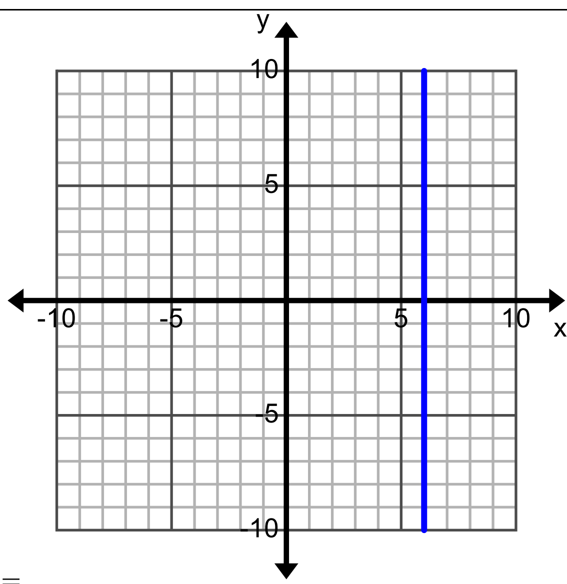
$m=$

8.



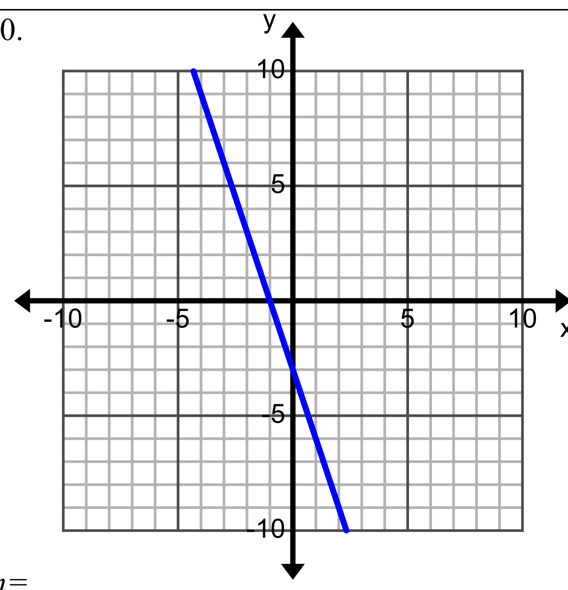
$m=$

9.



$m=$

10.

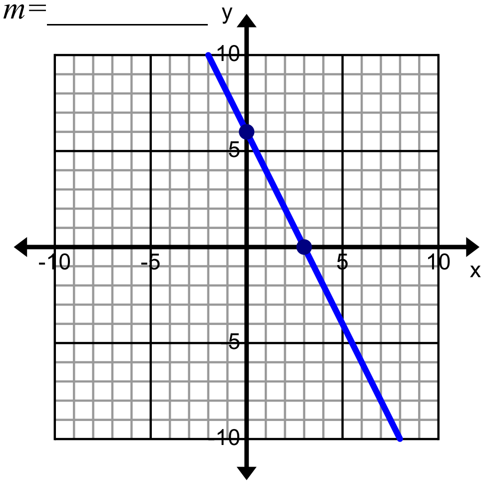


$m=$

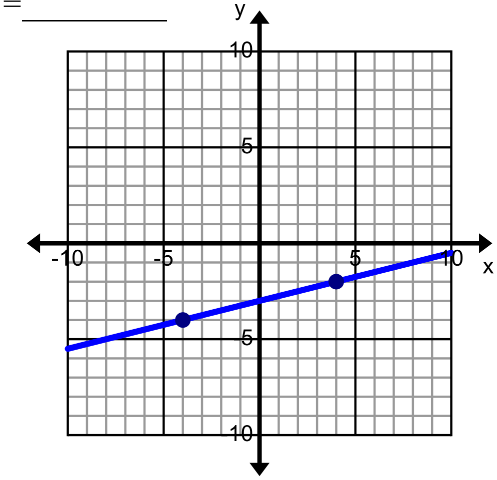
2.2e Class Activity: Finding Slope from Two Points

Calculate the slope of the following graphs:

1. $m=$____________



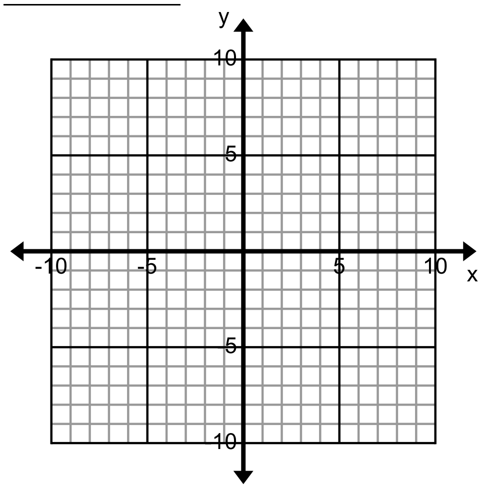
2. $m =$____________



Graph the following pairs of points. Use the graph to determine the slope.

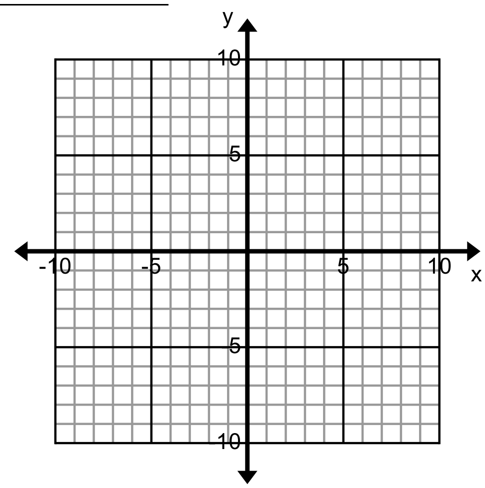
3. points: (4, 3) and (0, 1)

$m =$_____________



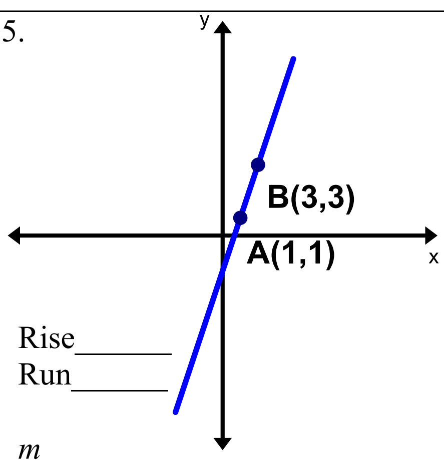
4. points: (1, 4) and (-2, 6)

$m =$_____________



Find the rise and run and slope of each line shown below. You will have to think of a way to use the coordinate points to find the rise and run.

5.

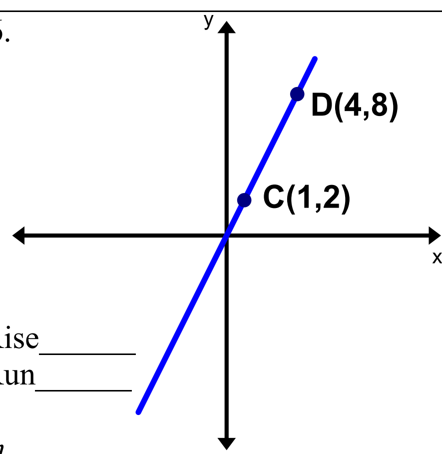


Rise_______

Run_______

m_________

6.

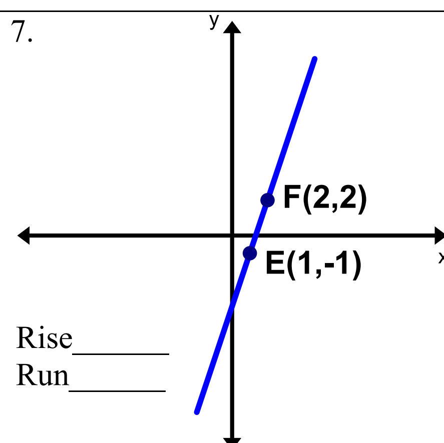


Rise_______

Run_______

m_________

7.

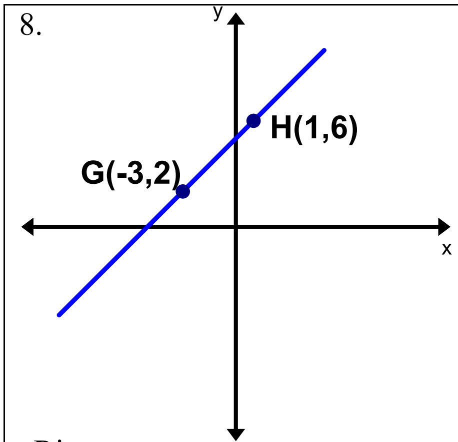


Rise_______

Run_______

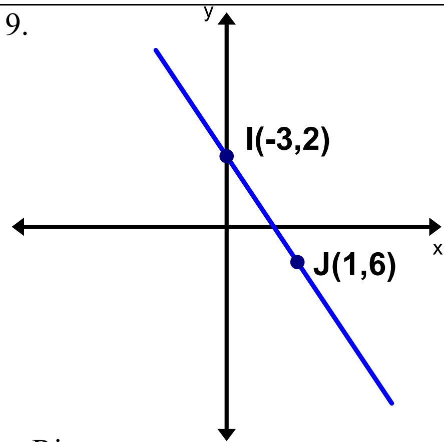
m_________

8.



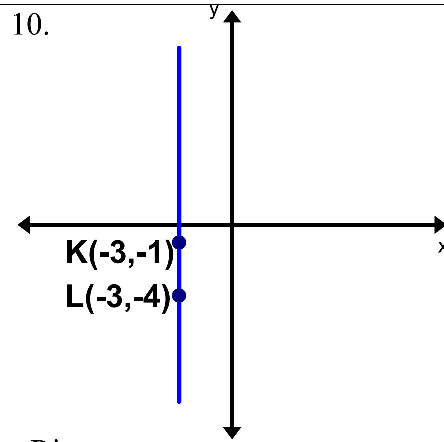
Rise______
Run______ m________

9.



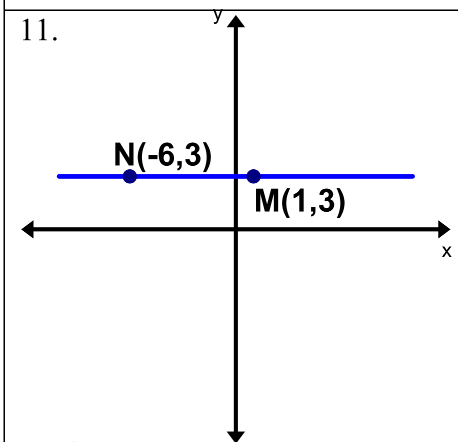
Rise______
Run______ m________

10.



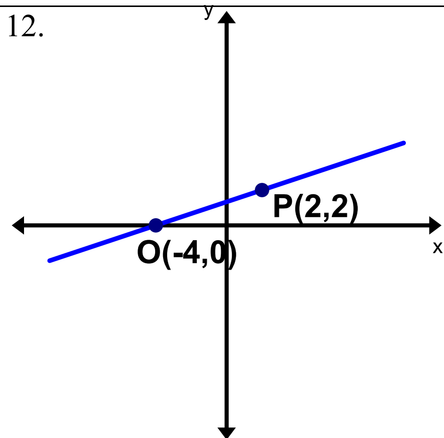
Rise______
Run______ m________

11.



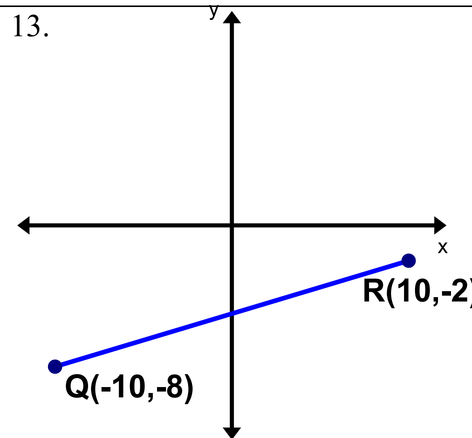
Rise______
Run______ m________

12.



Rise______
Run______ m________

13.



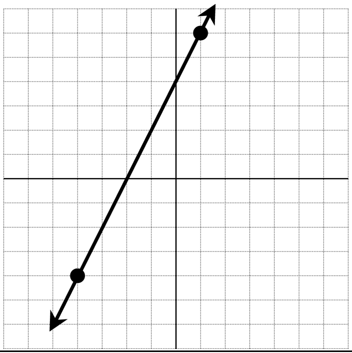
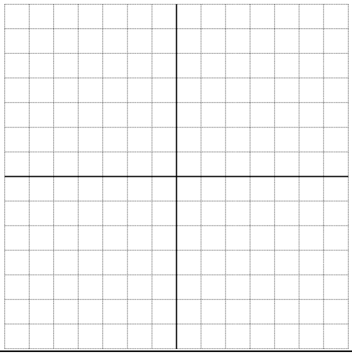
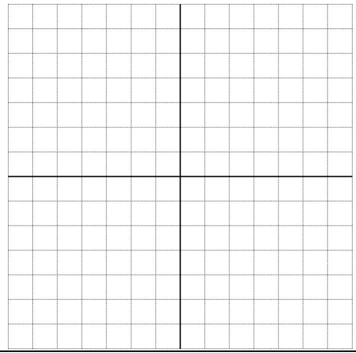
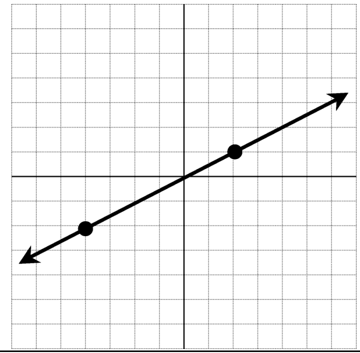
Rise______
Run______ m________

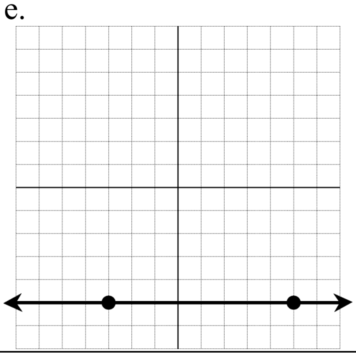
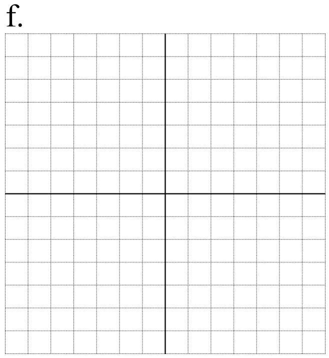
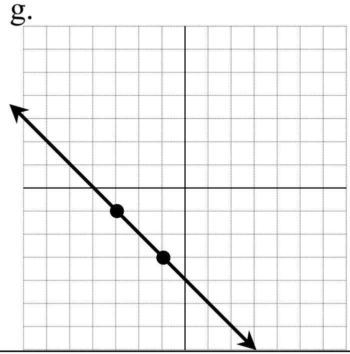
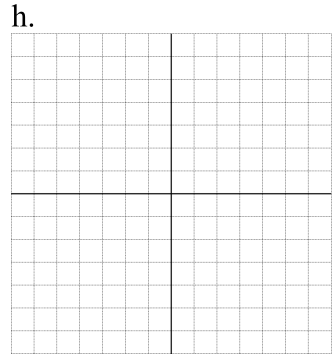
14. Graphing points can be time-consuming. Develop a procedure for calculating the slope without graphing each point. Explain your procedure below. Show that it works for problems 1-4 above.

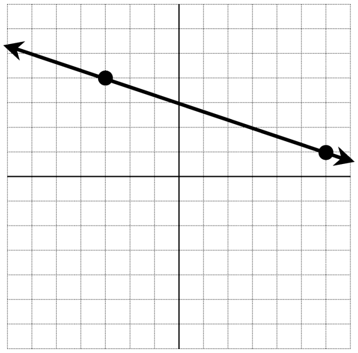
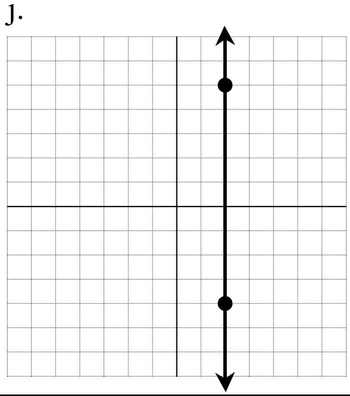
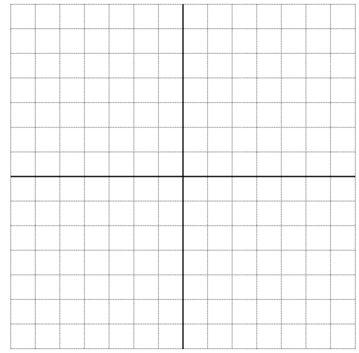
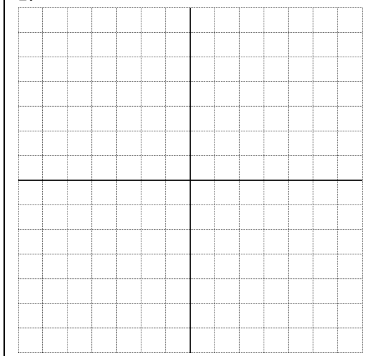
15. Discuss the methods for calculating slope without using right triangles on a graph. Write what you think about the methods.

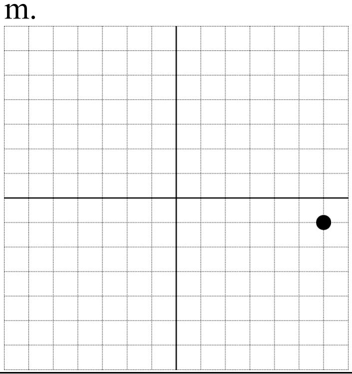
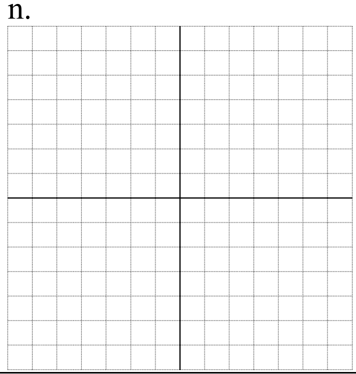
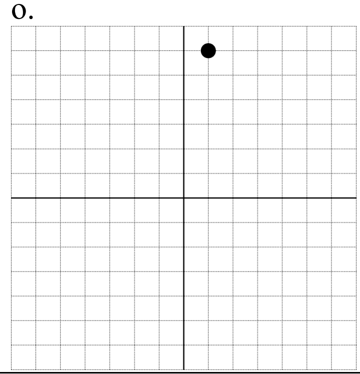
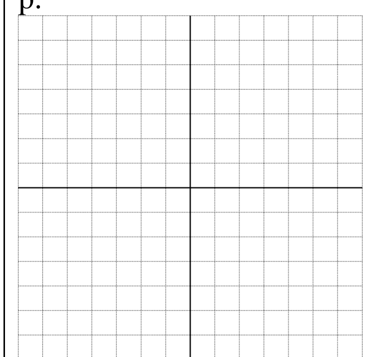
16. Now discuss this formula:. $m = \frac{y_2 - y_1}{x_2 - x_1}$ What does it mean? How does it work?

17. Fill in the missing information in the problems below. Use the empty box to calculate slope using the formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

a. 	b. 	c. 	d. 
(,) (,)	(-3, -6) (6. 0)	(4, -3) (-5, 9)	(,) (,)
$\frac{\Delta y}{\Delta x} = \_\_\_\_$	$\frac{\Delta y}{\Delta x} = \_\_\_\_$	$\frac{\Delta y}{\Delta x} = \_\_\_\_$	$\frac{\Delta y}{\Delta x} = \_\_\_\_$

e. 	f. 	g. 	h. 
(,) (,)	(-5 , -7) (-5, -3)	(,) (,)	(-1 , -7) (,)
$\frac{\Delta y}{\Delta x} = \_\_\_\_$	$\frac{\Delta y}{\Delta x} = \_\_\_\_$	$\frac{\Delta y}{\Delta x} = \_\_\_\_$	$\frac{\Delta y}{\Delta x} = 5$

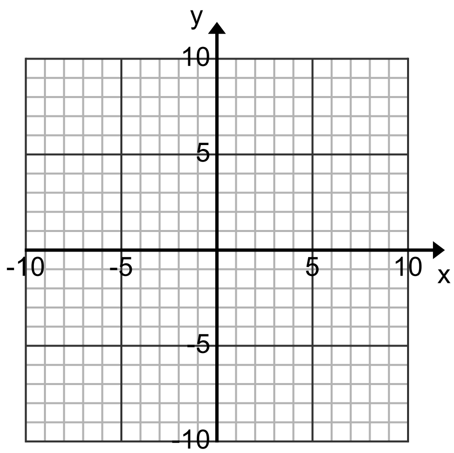
i. 	j. 	k. 	l. 
(,) (,)	(,) (,)	(-2 , -7) (-1 , -3)	(-7, 6) (6 , -7)
$\frac{\Delta y}{\Delta x} = \underline{\qquad}$	$\frac{\Delta y}{\Delta x} = \underline{\qquad}$	$\frac{\Delta y}{\Delta x} = \underline{\qquad}$	$\frac{\Delta y}{\Delta x} = \underline{\qquad}$

m. 	n. 	o. 	p. 
(-6 , 1) (,)	(-1, 6) (-4, 6)	(,) (,)	(0, -2) (,)
$\frac{\Delta y}{\Delta x} = \underline{\qquad}$	$\frac{\Delta y}{\Delta x} = \underline{\qquad}$	$\frac{\Delta y}{\Delta x} = \frac{3}{2}$	$\frac{\Delta y}{\Delta x} = \frac{-3}{4}$

2.2e Homework: Finding Slope from Two Points

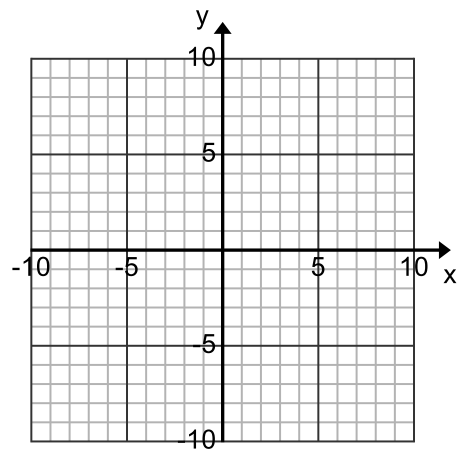
Graph the following pairs of points. Use the graph to determine the slope.

1. (-4, 3) and (2, 6)



Slope=___________________

2. (-1, 4) and (0, 1)



Slope=___________________

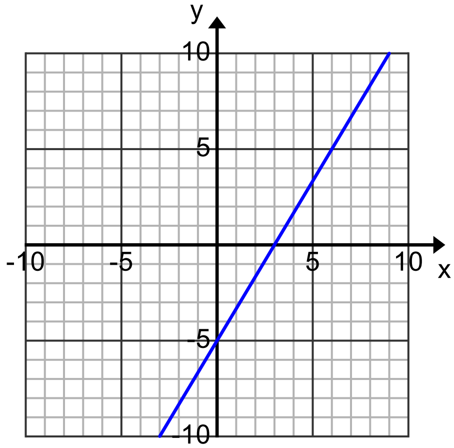
Calculate the slope of the line connecting each pair of points.

3. (1, 42) and (4, 40)
4. (-21, -2) and (-20, -5)
5. (3, -10) and (-6, -10)
6. (10, -11) and (11, -12)
7. (5, 1) and (-7, 13)
8. (14, -3) and (14, -7)
9. (8, 41) and (15, 27)
10. (17, 31) and (-1, -5)
11. (-5, 36) and (-4, 3)
12. (32, -23) and (-6, -2)

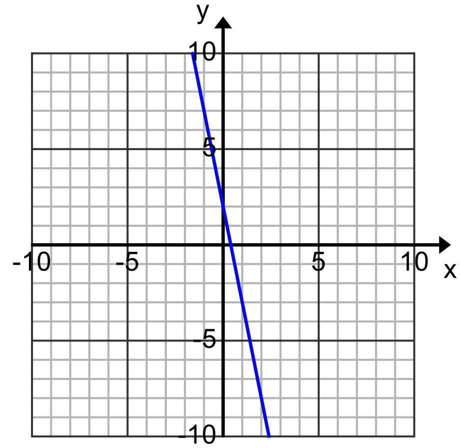
2.2f Class Activity: Practice Finding the Slope of a Line

Calculate the slope of the line on each graph.

1.
Slope=___________________



2.
Slope=_______________________



For each pair of points,

a. Calculate the slope of the line passing through each pair
b. Find one other point that lies on the line containing the given points.

3. (10, -6) and (-5, 4)

4. (7, 3) and (-3, 0)

5. (0, 4) and (1, 0)

6. (-5, 1) and (-5, -2)

Calculate the slope from each table. Calculate the slope twice, one time by using the Slope Formula with two points and the other time by finding the rate of change or unit rate in the table.

7.
Slope=____________

Slope=____________

x	y
3	4
4	5
5	6
6	7

8.
Slope=____________

Slope=____________

x	y
0	4
1	9
2	14
3	15

9.
Slope=____________

Slope=____________

x	y
0	9
3	12
6	15
9	18

10.
Slope=___________

Slope=____________

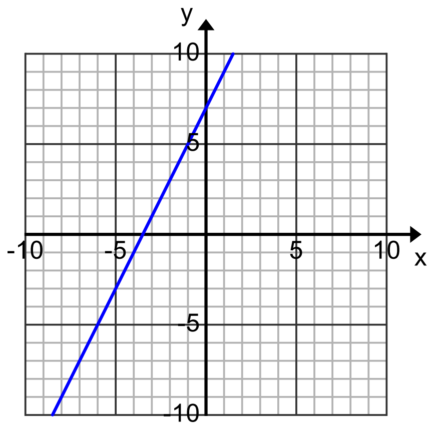
x	y
2	4
4	12
6	20
8	28

11. Why are the slopes the same no matter what two points you use to find the slope?

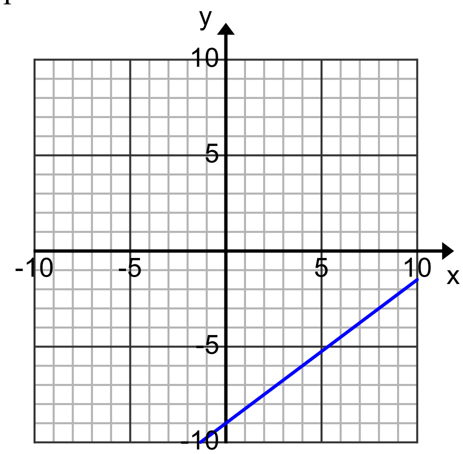
2.2f Homework: Practice Finding the Slope of a Line

Calculate the slope of the line on each graph.

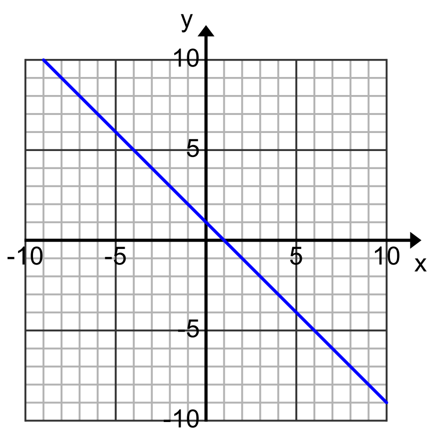
1. Slope =



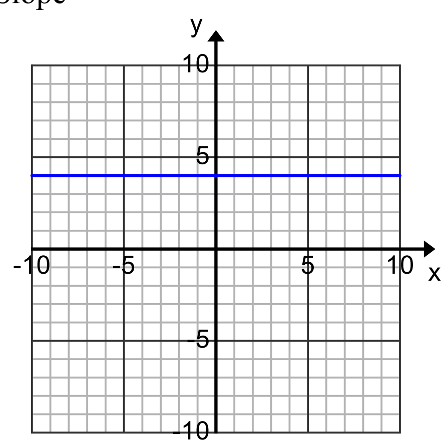
2. Slope =



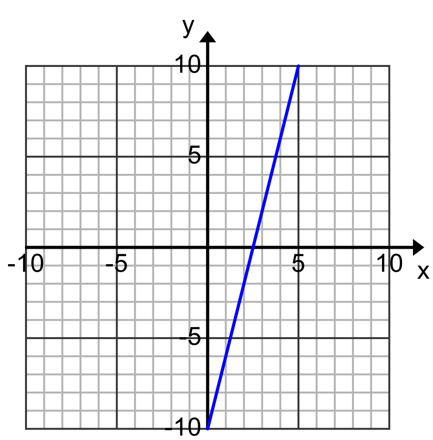
3. Slope =



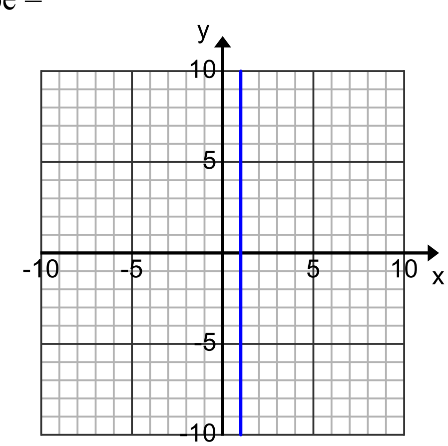
4. Slope =



5.Slope =



6. Slope =



Calculate the slope of the line passing through each pair of points.

7. (3, 9) and (4, 12)

8. (5, 15) and (6, 5)

9. (6, 9) and (18, 7)

10. (-8, -8) and (-1, -3)

For numbers 11 and 12;
a. Calculate the slope of the line passing through each pair
b. Find one other point that lies on the line containing the given points

11. (-6,-50 and (4,0)

12. (4,1) and (0,7)

Calculate the Slope from each table.

13. Slope =

x	*y*
3	-9
5	-15
9	-27

14. Slope =

x	*y*
8	1
6	3
2	7
-4	13

15. Slope =

x	*y*
0	4
1	-5
2	-14
3	-23

16. Slope =

x	*y*
10	1
8	1
-12	1
-14	1

17. Slope =

x	*y*
-5	2
5	6
10	8

18. Slope =

x	*y*
-3	5
-3	10
-3	15
-3	20

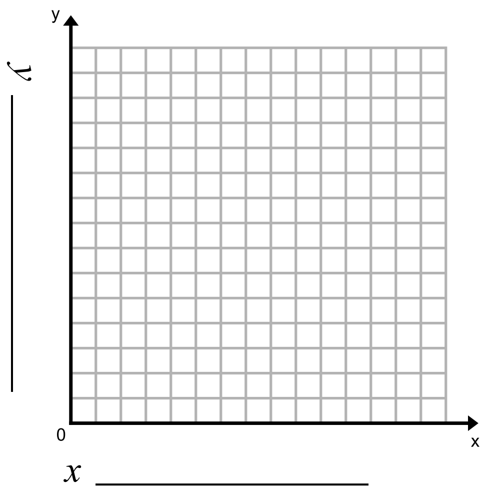
17. Why doesn't it matter which two points you use to find the slope?

2.2g Class Activity: Finding Slope from a Context

2. Gourmet jellybeans cost \$9 for 2 pounds.
   a. Complete the table.

Pounds	.5		3	4		8	10		20
Total Cost		\$9			\$27				

b. Label and graph axes. Graph the relationship.



c. What is the unit rate?

d. Write a sentence with correct units to describe the rate of change.

e. What is the slope of the line?

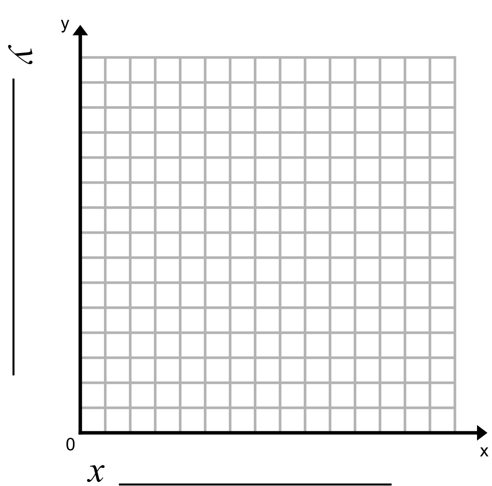
f. Write an equation to find the cost for any amount of jellybeans.

g. Why is the data graphed only in the first quadrant?

3. Kaelynn can solve 10 equations in 8 minutes.
   a. Complete the table.

Minutes	2		8			20	
Equations Solved		5		15			30

b. Label and graph the relationship.



c. What is the unit rate?

d. Write a sentence with correct units to describe the rate of change.

e. What is the slope of the line?

f. How are the slope of the line, unit rate and Kaelynn's rate of change all related?

g. Write an equation to find the equations solved for any number of minutes.

4. Mr. Irving and Mrs. Hendrickson pay babysitters differently.

b. Examine the table. Describe the difference.

Hours	Irving	Hendrickson
1	$9	$12
2	$18	$20
3	$27	$28
4	$36	$36
5	$45	$44

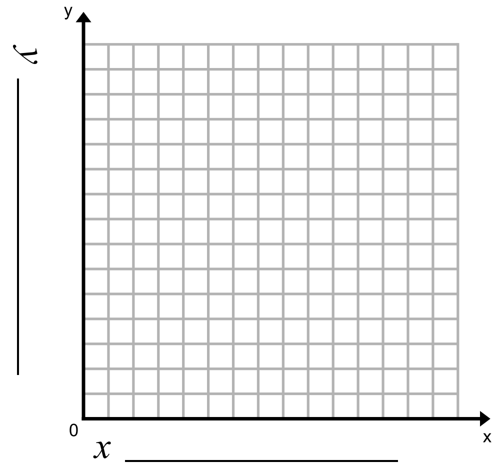
b. Are both relationships proportional?

c. Is there a unit rate for each? If so, what is it?

Irving? __________

Hendrickson? ________

d. Graph and label the two pay rates (two different lines).



e. What is the slope of the line?

Irving? __________ Hendrickson? ________

f. What does the slope tell you about these two situations?

g. Explain the different y intercepts.

h. Is one situation better than the other? Why or why not?

i. Write an equation for each situation.

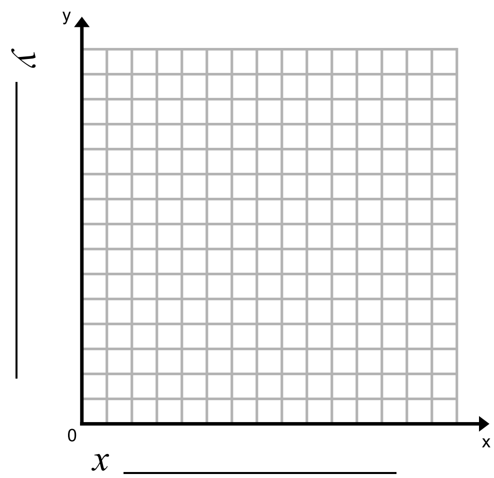
- Irving? __________
- Hendrickson? ________

2.2g Homework: Finding Slope from a Context

1. The soccer team is going out for hot dogs. Greg's Grill is having a special on hot dogs: four hot dogs for three dollars.
   a. Complete the table.

Hot Dogs	1			16		28	40
Total Cost		$3	$9		$18		

b. Label and graph the data given in the table.



c. What is the unit rate?

d. Write a sentence with correct units to describe the rate of change.

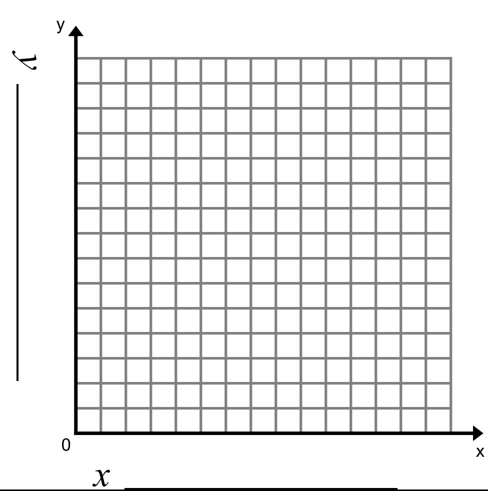
e. What is the slope of the line?

f. Write an equation to find the cost for any amount of hot dogs.

2. The state fair costs $2 to get in plus $.50 per ticket to go on rides. Complete the following table, showing the cost for getting into the fair with additional tickets for rides.
   a. Complete the table.

Tickets	0	1		10		30	
Total Expense			$4.50		$12		$20

b. Graph and label the relationship.



c. What is the rate of change?

d. What is the slope of the line?

e. Why does the line not pass through 0?

f. Write an equation to find the total expense at the fair with any amount of tickets purchased.

3. Excellent Bakers and Delicious Delights bakeries charge differently for sandwiches for business lunches.

a. Examine the table. Describe the difference.

Sandwiches	Excellent Bakeries	Delicious Delights
1	$20	$30
2	$40	$45
3	$60	$60
4	$80	$75
5	$100	$90

b. Are both relationships proportional?

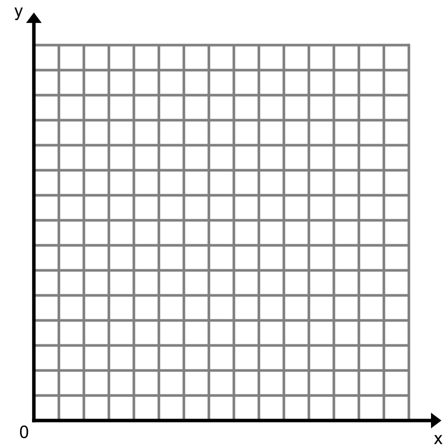
c. Is there a unit rate for each? If so, what is it?

Excellent _________

Delicious _______

b. Graph the two pay rates (two different lines). Label.

y ___________



x _________________

c. What is the slope of the line?

Excellent? _______ Delicious? _______

d. What does the slope tell you about these two situations?

e. Explain the different y intercepts.

f. Is one situation better than the other? Why or why not?

g. Write an equation for each situation.

- Excellent Bakery _________
- Delicious Delights _______

2.2h Self-Assessment: Section 2.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Deep Understanding, Skill Mastery
1. Show that the slope of a line can be calculated as rise/run for any two points on a line.			
2. Explain why the slope is the same between any two distinct points on a non-vertical line.			
3. Find the slope of a line from a graph			
4. Find the slope of a line from a set of points			
5. Find the slope of a line from a table			
6. Given a context, find slope from various starting points (2 points, table, line, equation).			

Section 2.3: Transition from Proportional Relationships to Linear Relationships

Section Overview:
A bridge from proportional relationships to linear relationships is achieved as students translate the graph of a proportional relation away from the origin and analyze that there is no effect on slope but that the relation is no longer proportional. The use of transformations to investigate this transition of proportional to linear is an integrated approach that uses the tools of translations to better understand proportional and linear graphs.

Concepts and Skills to Master:

1. Understand that a translation of the proportional equation $y=mx$ from the origin will produce a linear equation of the form $y=mx+b$
2. Recognize that m in $y=mx$ and $y=mx+b$ represents that proportional constant or slope of a line.
3. Begin to understand that b is where the line crosses the y-axis or is the y-intercept.
4. Explore the properties of translations and dilations: that the image of a line is a line parallel to it.

2.3a Class Activity: Proportional Relations to Linear Relations

Recall what we know about proportional relations.

- A proportional relationship is defined by a proportional constant
- When a proportional relationship is graphed it is a straight line going through the origin.
- A proportional relation can also be represented by an equation where the proportional constant relates your x value to your y value in the form $y=mx$ where m is the proportional constant or slope.
- The proportional constant is the same as a unit rate in a proportional relationship. Unit rate can be interpreted as the slope of a line.

We also know that not all relations are proportional. If a straight line does not go through the origin it does not have a proportional constant, however it does have a unit rate or slope. A geometric transformation can relate a proportional relationship to a non-proportional relationship.

In previous sections we used the geometric transformation called a dilation. Another type of transformation is called a translation. Shifting a line or moving all the points on the line the same distance and direction is a transformation is a **translation**.

For example, if you transform the line $y=x+4$ upwards by 3 units, every ordered pair that lies on that line gets moved up 3 units. Algebraically that means that you add 3 to every y value since this is a vertical shift.

$$(x,y)\rightarrow(x,y+3)$$

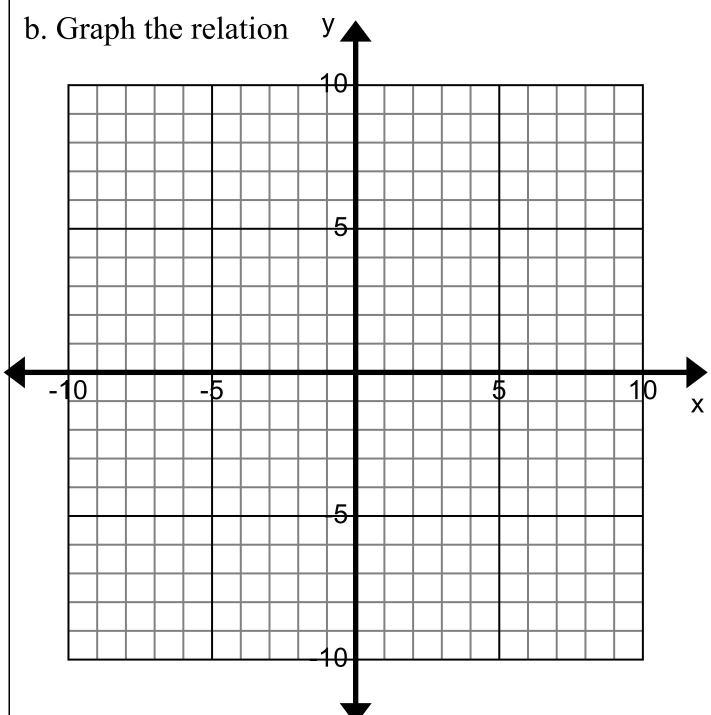
To confirm this, investigate this transformation below.

1. $y=x+4$

a. Make a table of values for the relation $y=x+4$

x	y

b. Graph the relation



c. Transform every point up 3 units and draw the new graph or the image in a different color.

d. Using your table of values add 3 to every y value. $(x,y)\rightarrow(x,y+3)$

x	y	$y+3$

e. Graph your new ordered pairs.

f. What do you observe about the transformation when you do it graphically and algebraically?

g. What do we know about the slopes of the lines for the pre-image and the image?

Translate the relations below to see how proportional and non-proportional relationships are related.

2. The sum of two numbers is 6.

a. Write an equation to represent the relationship. Let x and y represent the two numbers.

b. Fill in the table of values for x and y so that the equation is true.

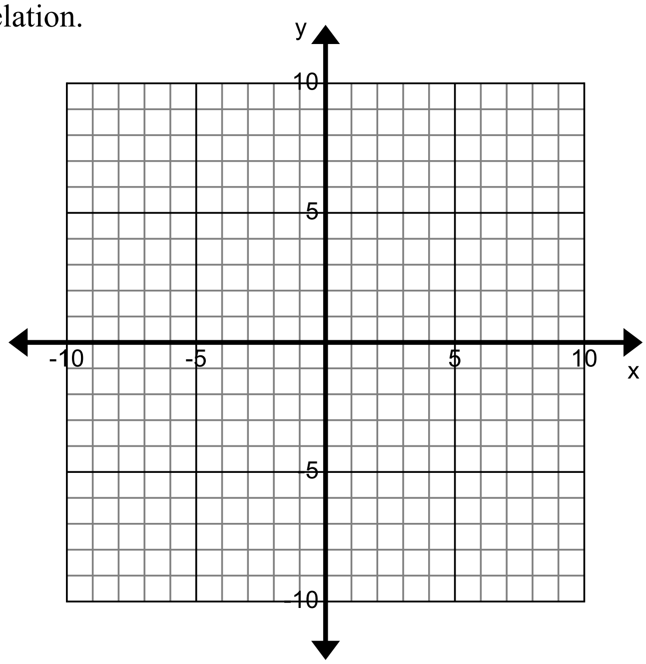
x					
y					

c. Are these the only values for x and y?

d. Find the ratios for these pairs of points to determine if the relationship is proportional. State whether this is a proportional relationship.

x					
y					
y/x					

e. Plot the points from your table to graph the relation.



f. Why do you connect your points on the graph?

g. How can you use the graph to determine if this is a proportional relationship?

h. What is the slope for this graph?

i. Using the coordinate plane on number 5 shift the graph down 6 units (move every point down 6 units) and graph a new line. You could also move every point to the left 6 units and you will get the same line.

$$(x, y) \to (x, y-6)$$

j. What is the equation of this line? (Think about how you would change the original equation based off of how you shifted the graph)

k. Make a table of values for this relationship

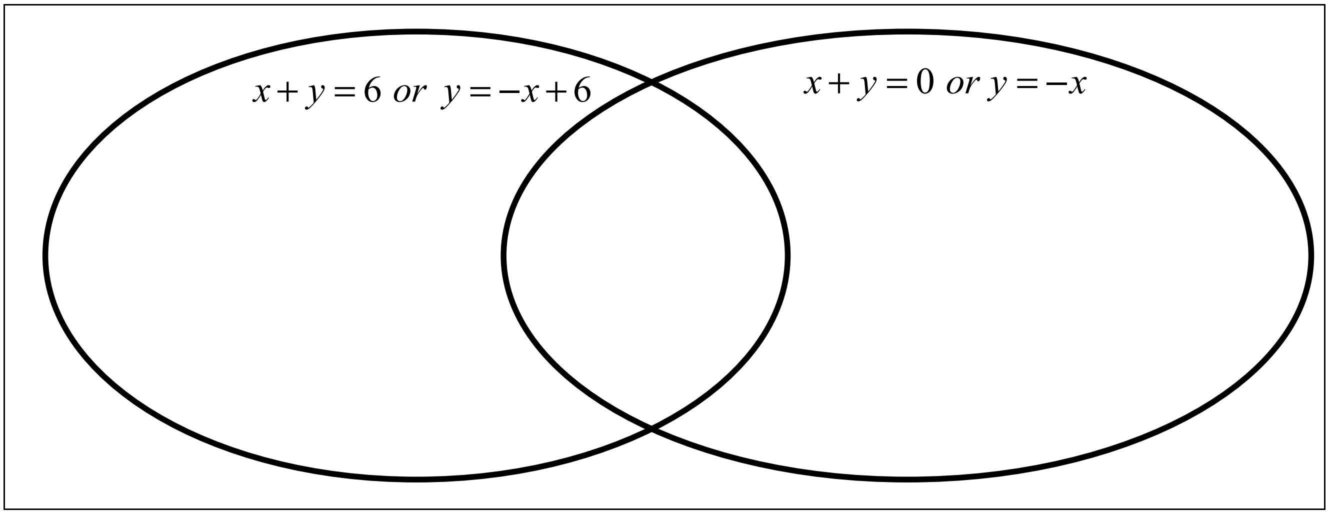
x					
y					
y/x					

l. What is the slope for this graph?

m. What similarities do the two lines and equations have?

n. Use the new graph and table to determine if this is a proportional relationship? If so what is the proportional constant?

4. Use the information above to compare and contrast the two relations using the Venn Diagram below. Be sure to list similarities and differences about their graphs and equations.

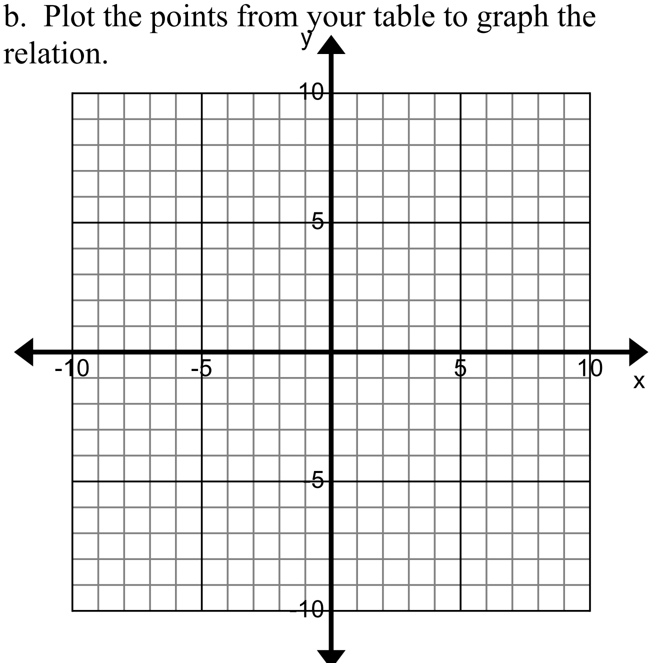


5. $y=3x+4$

a. Fill in the table of values for x and y for the relation.

x					
y					
y/x					

b. Plot the points from your table to graph the relation.



c. Use the table and the graph to determine if this relationship is proportional. Justify your answer.

d. Is this relation linear? (Does it make a straight line)

e. What is the slope for the graph?

f. Determine how you would shift the graph so that it will pass the origin. Then graph the new line on the coordinate plane.

g. What is the equation of the proportional relation that coincides with this shift.(Think about how the original equation would change based off of how you shifted the graph.

h. Make a table of values for this relationship

x					
y					
y/x					

i. What is the slope for this graph?

j. What similarities do the two lines and equations have?

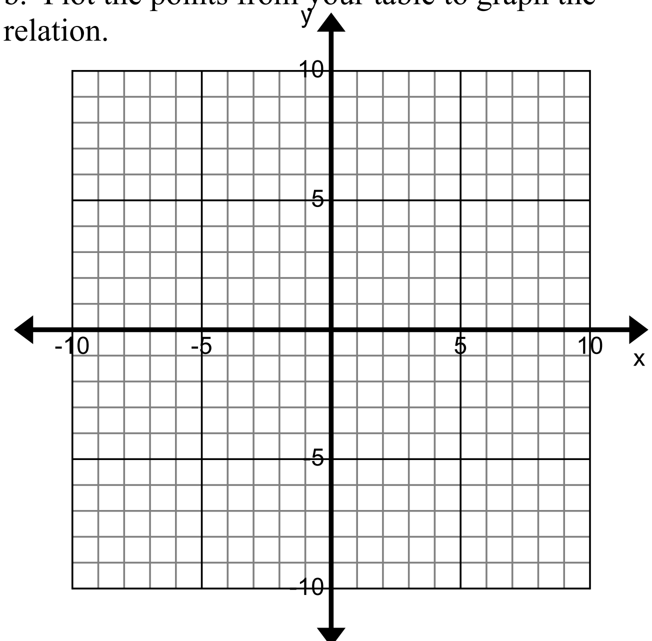
k. Use the new graph and table to determine if this is a proportional relationship? If so what is the proportional constant?

6. $y = \frac{1}{2}x - 2$

a. Fill in the table of values for x and y for the relation.

x					
y					
y/x					

b. Plot the points from your table to graph the relation.



c. Use the table and the graph to determine if this relationship is proportional. Justify your answer.

d. Is this relation linear?

e. What is the slope for the graph?

f. Determine how you would shift the graph so that it will pass the origin. Then graph the new line on the coordinate plane.

g. What is the equation of the proportional relation that coincides with this shift.(Think about how the original equation would change based off of how you shifted the graph)

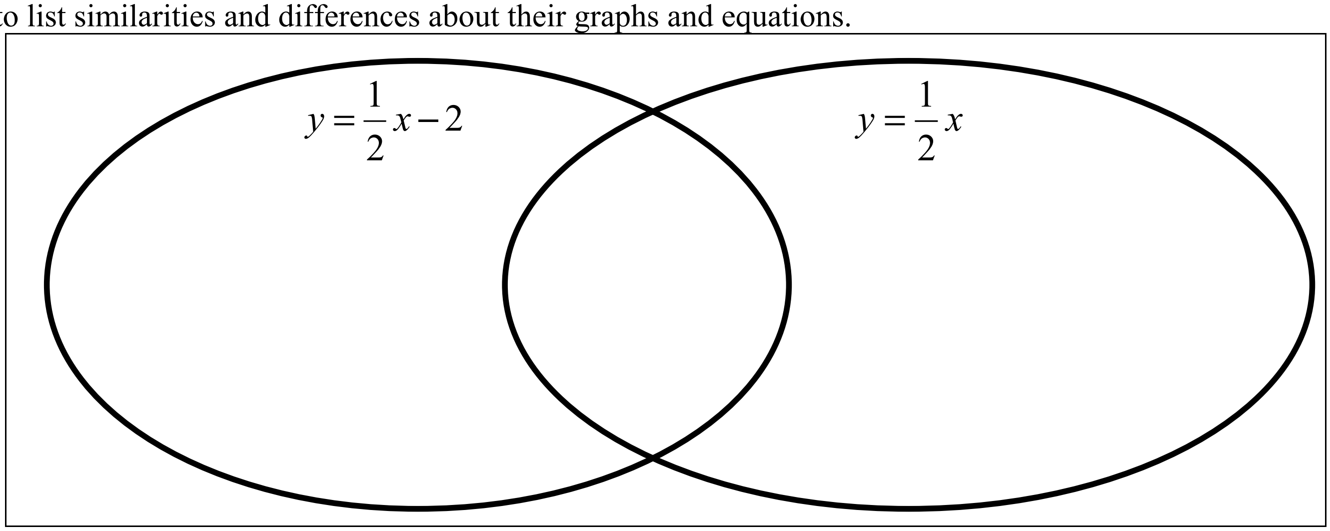
h. Make a table of values for this relationship

x					
y					
y/x					

i. What is the slope for this graph?

j. Use the new graph and table to determine if this is a proportional relationship? If so what is the proportional constant?

7. Use the information above to compare and contrast the two relations using the Venn Diagram below. Be sure to list similarities and differences about their graphs and equations.



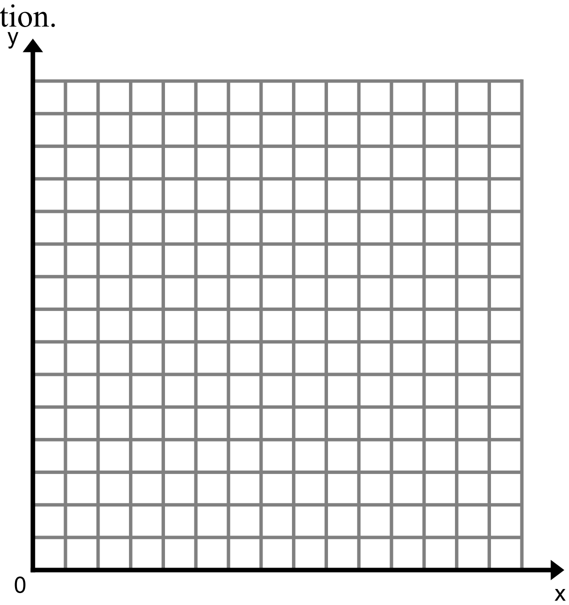
8. The product of two numbers is 6

a. Write an equation to represent the relationship. Let x and y represent the two numbers.

b. Fill in the table of values for x and y for the relation. (For this relation only use positive numbers)

x	1	2	3	4	5
y					
y/x					

c. Plot the points from your table to graph the relation.



d. Can you find the slope of this line? Why or why not?

e. Use the table and the graph to determine if this relationship is proportional. Justify your answer.

f. Is this relation linear?

Not all relations display a straight line when graphed. We say they are not "linear".

Answer the following questions below to summarize your thinking.

9. What do the graphs of all proportional relationships have in common?

10. When graphs make a continuous straight line we say they are "Linear". Do all proportional relationships generate a graph that is linear?

11. Are all linear graphs proportional?

12. How can you tell if a linear graph is not proportional?

13. Fill in the blanks using the words *proportional* or *linear*
"All proportional relationships are ____________________but not all linear relationships are ______________________."

Reflect on the Venn Diagrams that you filled out to answer the following questions.

14. What form of an equation did all of the proportional relationships have?

15. What form of an equation did all of your linear non-proportional equations have?

16. How are these equations related to each other? (Think about how you found the equation of the image based off of the pre-image.)

17. How are the lines of the pre-image and image related to each other?

2.3a Homework: Proportional Relations to Linear Relations
Complete the following tasks for each given relation.

1. The difference of two numbers is 2.

a. Write an equation to represent the relationship. Let x and y represent the two numbers.

b. Fill in the table of values for x and y so that the equation is true.

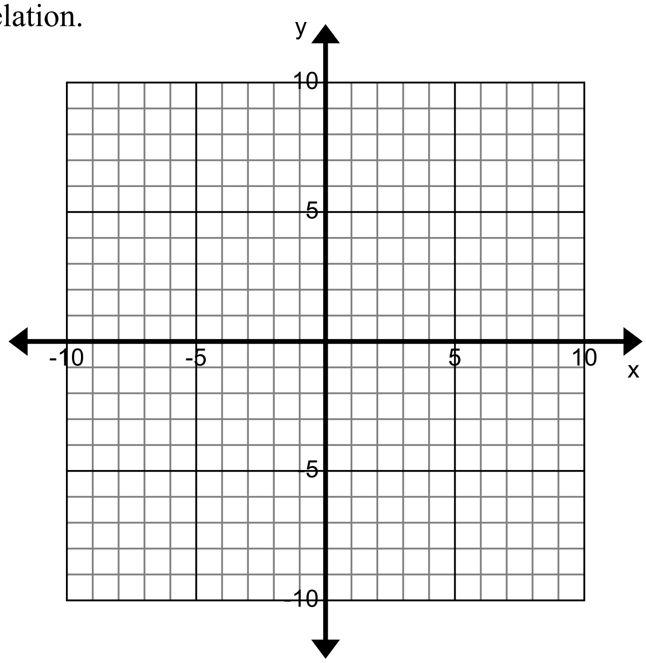
x					
y					

c. Are these the only values for x and y?

d. Find the ratios for these pairs of points to determine if the relationship is proportional. State whether this is a proportional relationship.

x					
y					
y/x					

e. Plot the points from your table to graph the relation.



f. Why do you connect your points on the graph?

g. How can you use the graph to determine if this is a proportional relationship?

h. What is the slope for this graph?

i. Using the coordinate plane on number 5 shift the graph up 2 units (move every point up 2 units) and graph a new line. You could also move every point to the left 2 units and you will get the same line.

$$(x, y) \rightarrow (x, y+2)$$

j. What is the equation of this line? (Think about how you would change the original equation based off of how you shifted the graph)

k. Make a table of values for this relationship

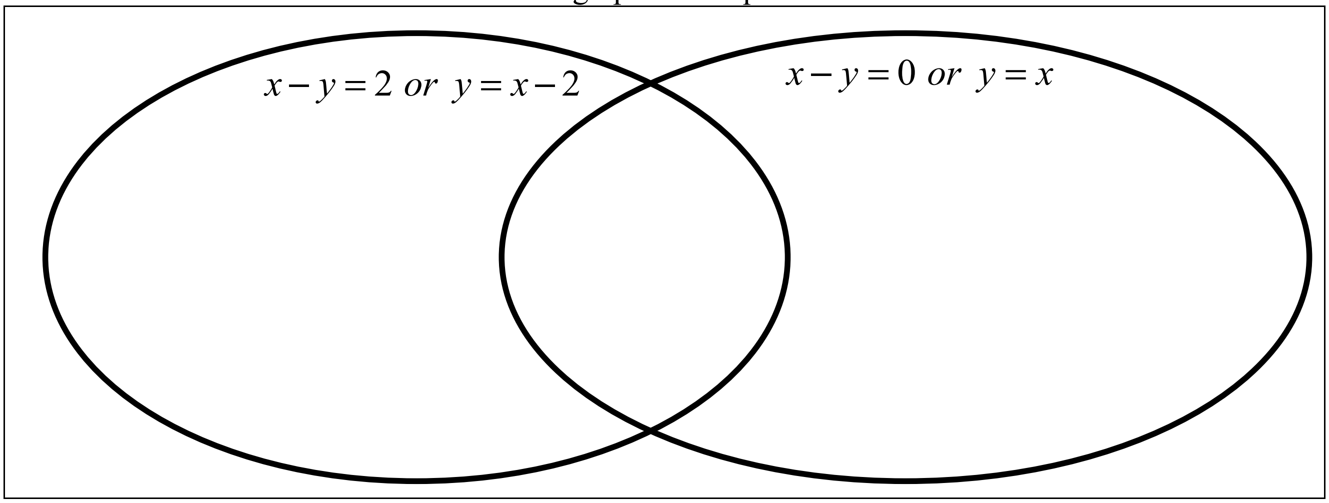
x					
y					
y/x					

l. What is the slope for this graph?

m. What similarities do the two lines and equations have?

n. Use the new graph and table to determine if this is a proportional relationship? If so what is the proportional constant?

2. Use the information above to compare and contrast the two relations using the Venn Diagram below. Be sure to list similarities and differences about their graphs and equations.

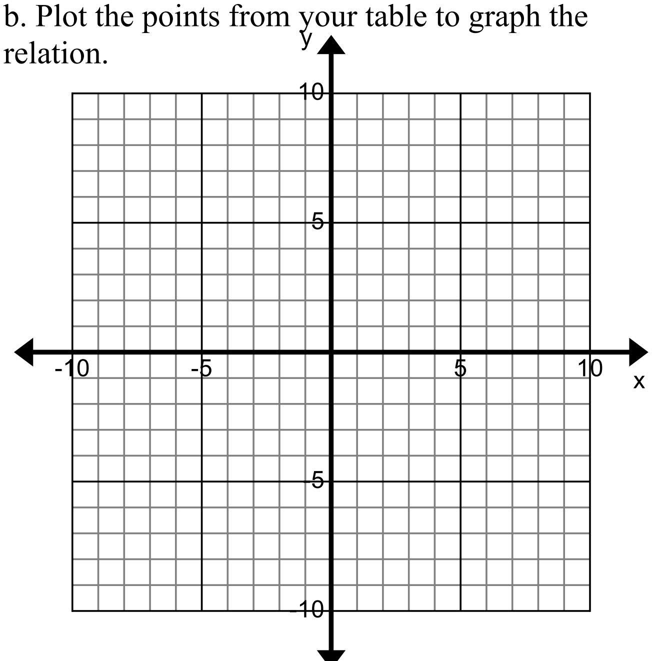


3. $y = -2x + 4$

a. Fill in the table of values for x and y for the relation.

x					
y					
y/x					

b. Plot the points from your table to graph the relation.



c. Use the table and the graph to determine if this relationship is proportional. Justify your answer.

d. Is this relation linear? (Does it make a straight line)

e. What is the slope for the graph?

f. Determine would you shift the graph so that it will pass the origin. Then graph the new line on the coordinate plane.

f. What is the equation of the proportional relation that coincides with this shift.(Think about how the original equation would change based off of how you shifted the graph)

g. Make a table of values for this relationship

x					
y					
y/x					

h. What is the slope for the image?

i. What similarities do the two lines and equations have?

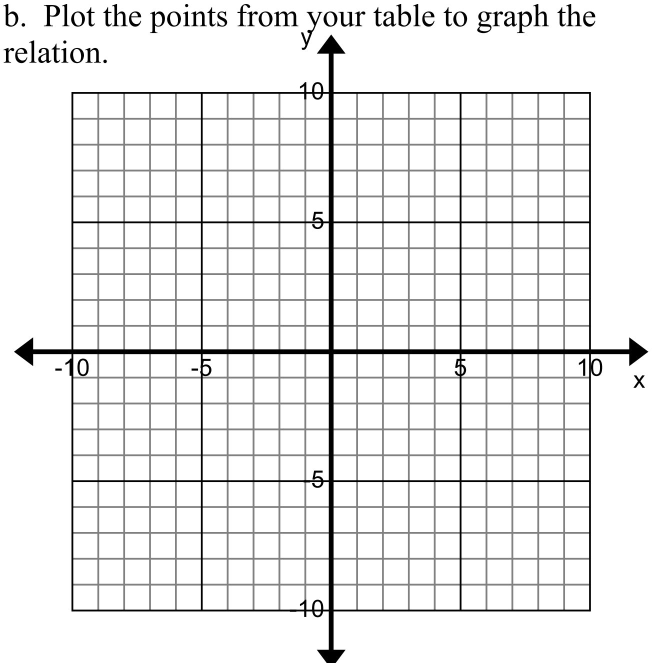
j. Use the new graph and table to determine if this is a proportional relationship? If so what is the proportional constant?

4. $y = \frac{2}{3}x - 1$

a. Fill in the table of values for *x* and *y* for the relation.

x					
y					
y/x					

b. Plot the points from your table to graph the relation.



c. Use the table and the graph to determine if this relationship is proportional. Justify your answer.

d. Is this relation linear?

e. What is the slope for the graph?

f. Determine would you shift the graph so that it will pass the origin. Then graph the new line on the coordinate plane.

g. What is the equation of the proportional relation that coincides with this shift.(Think about how the original equation would change based off of how you shifted the graph)

h. Make a table of values for this relationship

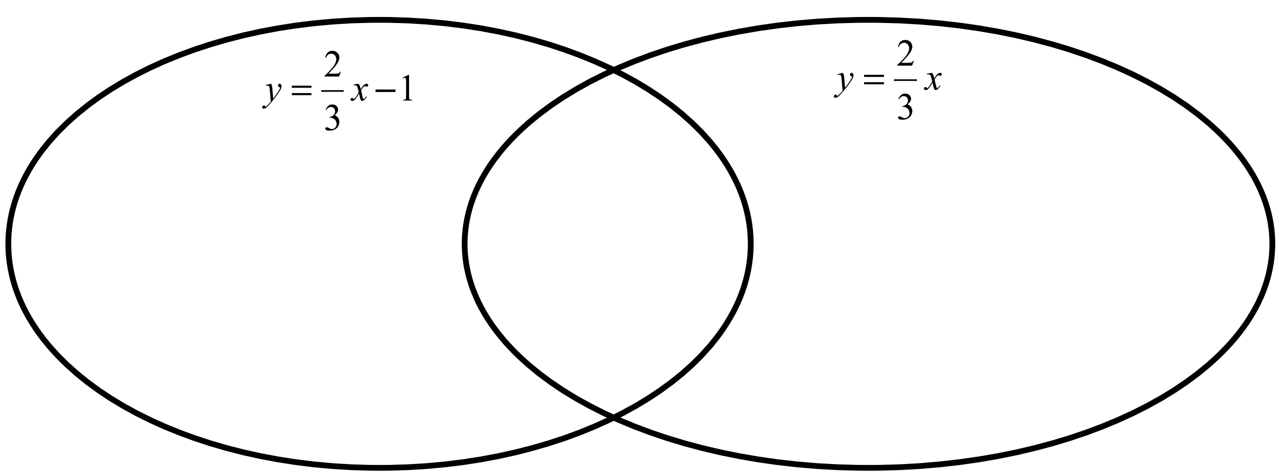
x					
y					
y/x					

i. What is the slope for the image?

j. What similarities do the two lines and equations have?

k. Use the new graph and table to determine if this is a proportional relationship? If so what is the proportional constant?

5. Use the information above to compare and contrast the two relations using the Venn Diagram below. Be sure to list similarities and differences about their graphs and equations.

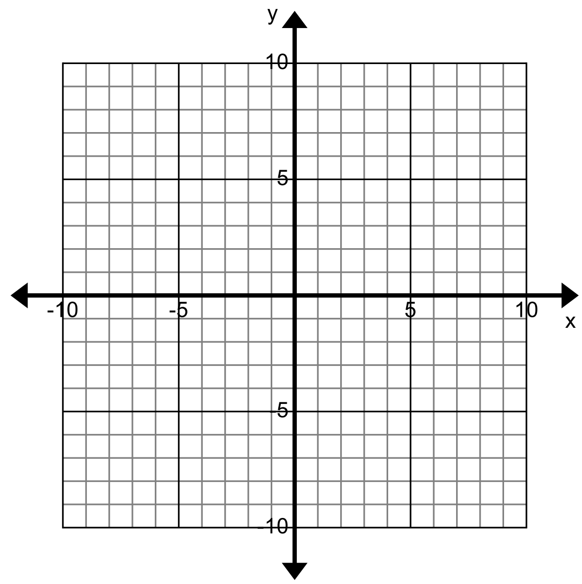


6. $y = x^2$

a. Fill in the table of values for x and y for the relation.

x	-2	-1	0	1	2
y					
y/x					

b. Plot the points from your table to graph the relation.



c. Use the table and the graph to determine if this relationship is proportional. Justify your answer.

d. Can you find the slope of this line? Why or why not?

e. Is this relation linear?

Answer the following questions below to summarize your thinking.

7. How do you know if a relation is proportional based off of its graph?

8. How do you know if a graph is linear?

9. How do you know if a relation is proportional based off of a set of ordered pairs or a table?

10. How do you know if a relation is proportional based off of an equation?

11. Given the following two forms of an equation label each one as *Linear Proportional* or *Linear Non-Proportional*

$y=mx$:

$y=mx+b$:

12. What do you think the m stands for in the equation?

13. What do you think that the b stands for in the equation?

14. Explain in your own words how a translation transitions a line that represents a proportional relationship to one the represents a non-proportional relationship. Be sure to include a discussion about their equations as well.

2.3b Self-Assessment: Section 2.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Deep Understanding, Skill Mastery
1. Understand that a translation of the proportional equation $y=mx$ from the origin will produce a linear equation of the form $y=mx+b$			
2. Recognize that m in $y=mx$ and $y=mx+b$ represents that proportional constant or slope of a line.			
3. Begin to understand that b is where the line crosses the y-axis or is the y-intercept.			
4. Explore the properties of translations and dilations: that the image of a line is a line parallel to it.			