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Chapter 4: Functions (4 weeks)

Utah Core Standard(s):

- Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (8.F.1)
- Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line. (8.F.3)
- Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (8.F.4)
- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change. (8.F.2)
- Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (8.F.5)

Academic Vocabulary: function, input, output, independent variable, dependent variable, linear, nonlinear, increasing, decreasing, constant, discrete, continuous, intercepts

Chapter Overview: In this chapter, the theme changes from that of solving an equation for an unknown number, to that of "function" that describes a relationship between two variables. In a function, the emphasis is on the relationship between two varying quantities where one value (the output) depends on another value (the input). We start the chapter with an introduction to the concept of function and provide students with the opportunity to explore functional relationships algebraically, graphically, numerically in tables, and through verbal descriptions. We then make the distinction between linear and nonlinear functions. Students analyze the characteristics of the graphs, table, equations, and contexts of linear and nonlinear functions, solidifying the understanding that linear functions grow by equal differences over equal intervals. Finally, students use functions to model relationships between quantities that are linearly related.

Connections to Content:
Prior Knowledge: Up to this point, students have been working with linear equations. They know how to solve, write, and graph equations. In this chapter, students make the transition to function. In the realm of functions, we begin to interpret symbols as variables that range over a whole set of numbers. Functions describe situations where one quantity determines another. In this chapter, we seek to understand the relationship between the two quantities and to construct a function to model the relationship between two quantities that are linearly related.

Future Knowledge: This chapter builds an understanding of what a function is and gives students the opportunity to interpret functions represented in different ways, identify the key features of functions, and construct functions for quantities that are linearly related. This work is fundamental to future coursework where students will apply these concepts, skills, and understandings to additional families of functions.

MATHEMATICAL PRACTICE STANDARDS (emphasized):

<table>
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<td>Make sense of problems and persevere in solving them.</td>
<td>On Tamara’s first day of math class, her teacher asked the students to shake hands with everyone in the room to introduce themselves. There are 26 students total in the math class. Can you determine the number of handshakes that took place in Tamara’s math class on the first day of class? Can the number of students vs. the number of handshakes exchanged be modeled by a linear function? Justify your answer.
As students grapple with this problem, they will start to look for entry points to its solution. They may consider a similar situation with fewer students. They may construct a picture, table, graph, or equation. They may even act it out, investigating the solution with a concrete model. Once they have gained entry into the problem, students may look for patterns and shortcuts that will help them to arrive at a solution either numerically or algebraically.</td>
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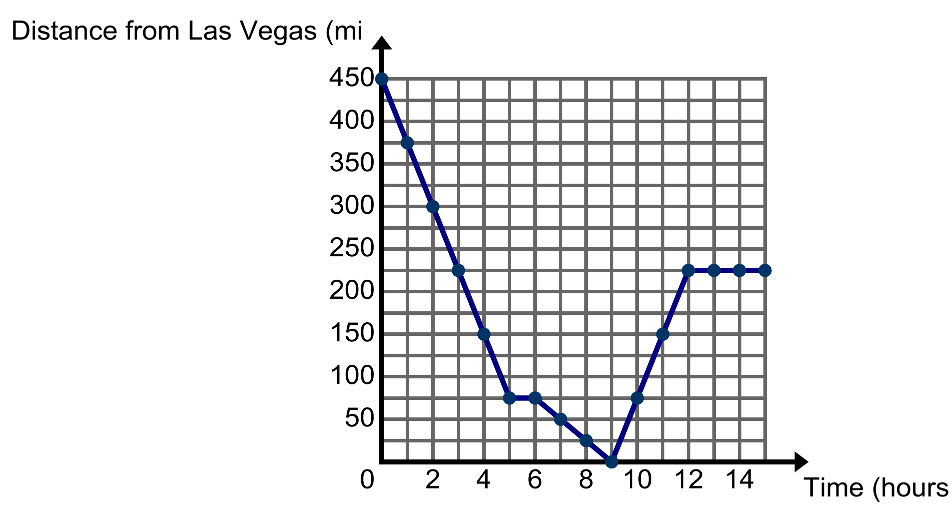
<td>Model with mathematics.</td>
<td>Throughout this chapter, students will apply the mathematics they have learned to solve problems arising in everyday life, society, and workplace. The following problems give students the opportunity to use functions to model relationships between two quantities.
Steve is a lifeguard at a local community pool. Each day at noon, he records the temperature and the number of people in the pool. Do you think the relationship between temperature and number of people in the pool is linear? Why or why not?

Two thousand, five hundred students attend a local high school. School starts at 8 am and ends at 2:30 pm. Many students stay after school for clubs, sports, etc. The school has a one-hour lunch at noon and seniors are allowed to leave campus for lunch. Sketch a graph of the number of cars in the student parking lot from 6 am to 4 pm.

Ben and his family took a road trip to visit their cousins. The graph below shows their journey. Label the key features of the graph. Write a story about the graph.



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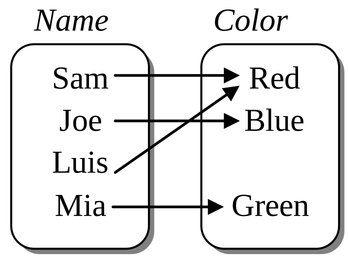
<td>Attend to precision.</td>
<td>Functions can also be described by non-numeric relations. A mapping is a representation of a function that helps to better understand non-numeric relations. Study each non-numeric relation and its mapping below. Then decide if the relation represents a function. Explain your answer.

Student’s name versus the color of their shirt.



As students analyze relations represented in mappings they must attend to the details given by the arrows and their direction. Precision is required as they determine if the arrows are showing one unique output for every input. They must be very careful as they analyze the correspondence between the input and output.</td>
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<td>Look for and express regularity in repeated reasoning.</td>
<td>Directions: Determine whether the situations you choose can be modeled by a linear function or not. Provide evidence to support your claim. Show your work in the space below.

Mr. Cortez drove at a constant rate for 5 hours. At the end of 2 hours he had driven 90 miles. After 5 hours, he had driven 225 miles. Can time vs. distance driven be modeled by a linear function? Provide evidence to support your claim.

Round 1 of a tennis tournament starts with 64 players. After each round, half the players have lost and are eliminated from the tournament. Therefore, in round 2 there are 32 players, in round 3 there are 16 players and so on. Can round number vs. number of players be modeled by a linear function? Provide evidence to support your claim.

In these problems, students must determine whether the relationship between the two quantities can be modeled by a linear function. This will require them to pay attention to how the quantities are growing (changing) in relation to each other. They will distinguish between the rates of change of linear and nonlinear functions, solidifying that linear functions grow by equal differences over equal intervals.</td>
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<td>Construct viable arguments and critique the reasoning of others.</td>
<td>Compare and contrast the relationship of the gumball machines at Vincent Drug and Marley’s Drug Store. If needed revise your conjecture about what kind of relationship makes a function and what disqualifies a relationship from being a function.
As students create, modify, and formulate their definition of a function they are constructing a viable argument that describes their thoughts on what a function is and what it is not. They make conjectures and build a logical progression of statements to explore the truth about their conjectures. They can share their definitions with others and decide whether they make sense and compare others’ thoughts and ideas to their own.</td>
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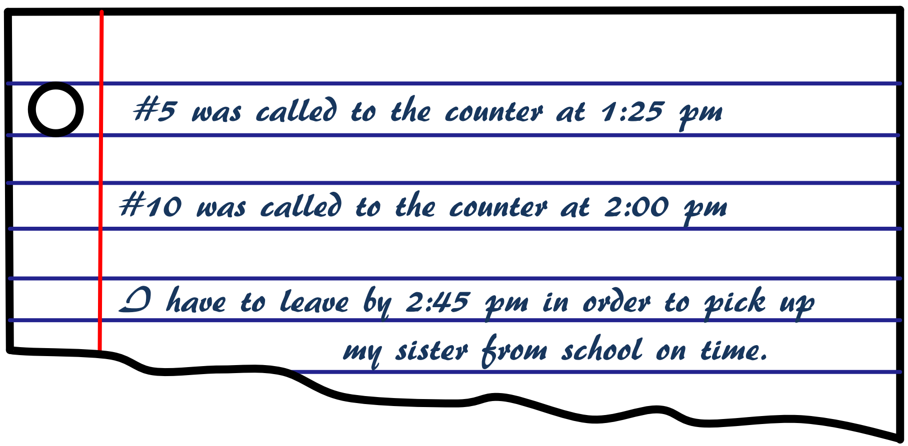
4.0: Anchor Problem

The DMV

1. Nazhoni has completed her Driver's Education Training and is at the DMV (Division of Motor Vehicles) waiting in line to get her license to drive. She entered the DMV at 12:50 and pulled a number 17 to reserve a spot in line. Nazhoni notices that all of the employees at the DMV are still at lunch when she arrives. Once the employees return they start with number 1. There is digital sign showing the number for the person who is at the counter being helped.

Nazhoni jots down some information on a piece of scratch paper as she is waiting in line.



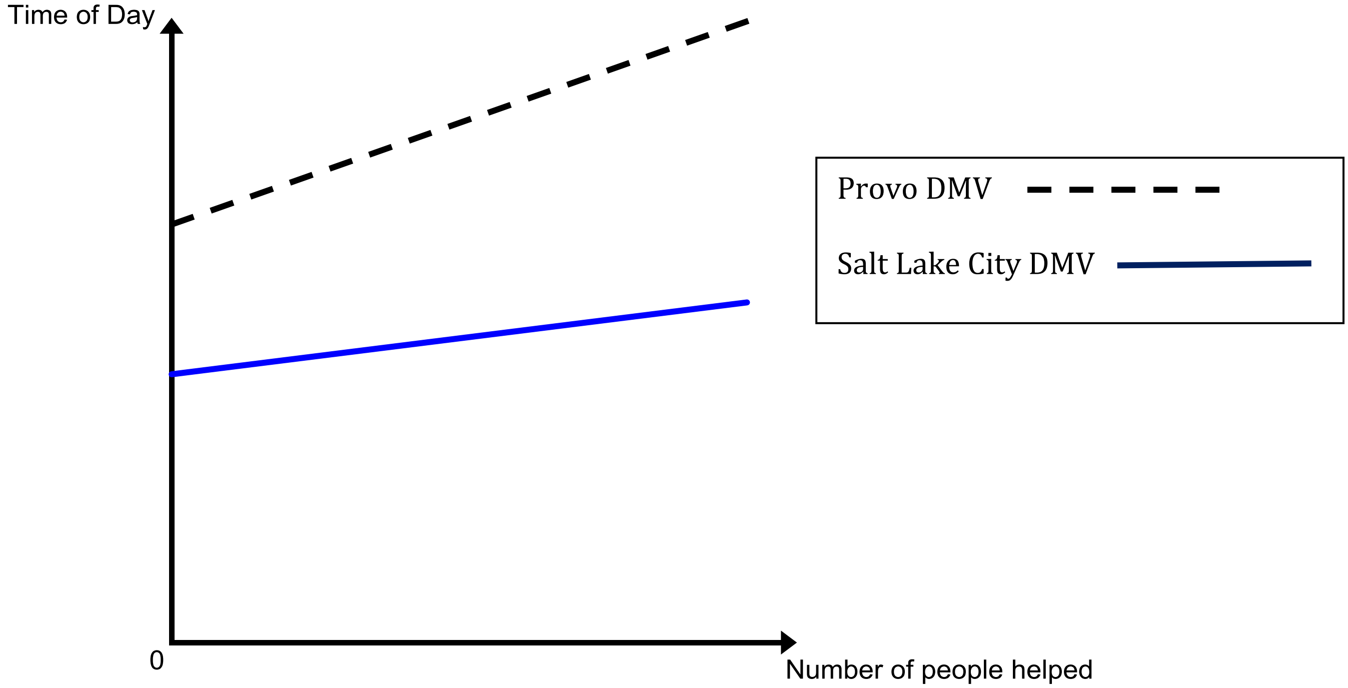
a. Use the picture of the scratch paper above to estimate what time it will be when Nazhomi will make it to the front of the line. **(Note: Assume that each person takes the same amount of time while being helped at the counter)**

b. Will Nazhoni make it to the front of the line in time to pick up her sister from school?

c. Estimate at what time the employees returned from lunch and began working.

d. Write an equation that a person waiting in line, with any number, on this particular afternoon could use to estimate how many minutes they will be at the DMV if they enter after 12:50.

2. The DMV in Provo and Salt Lake opened their doors for the day at the same time. The graphs below show the time of day as a function of the number of the person called to the counter. Write down as many differences between the two DMVs as you can based upon the graphs.



3. Do you think it is realistic that it takes the exact same amount of time for each person at the DMV? Explain.

4. The following table shows more realistic data for the waiting time at the DMV. Is there a constant rate of change for this data? If not, is the data still useful? What can be inferred about the information given from the table?

Time	# Being Helped
12:58	30
1:25	33
2:00	37
2:08	38
2:50	44
3:30	49

This problem was adapted from a task on Illustrative Mathematics.

Section 4.1: Define Functions

Section Overview:
This section begins by using a context to introduce a relation that represents a function and one that is not a function. By analyzing several contextual examples students derive their own definition of a function. They also write their own examples of relations that generate functions and examples that do not represent a function. In the next section the candy machine analogy is used to help students further their understanding of a function and input and output values. Students briefly write functions from the function machine but will formally build and model functions in later sections. Non-numeric functions are also touched on but will be studied in more depth in Secondary I. In section 4.1c students must determine if a relation is a function from different representations (i.e. table, graph, mapping, story, patterns, equations, and ordered pairs). Finally an in-depth study of independent and dependent variables is given in the last section as students use their understanding of functional relationships to determine how two quantities are related to one another.

Concepts and Skills to be Mastered:
By the end of this section students should be able to:

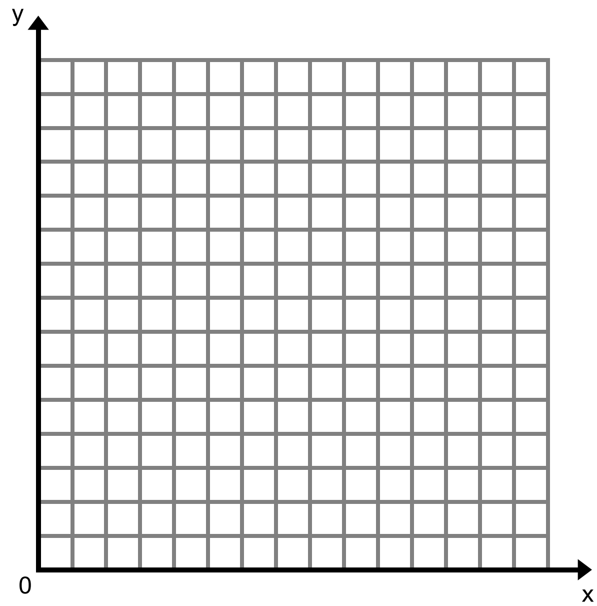
1. Understand that a function is a rule that assigns to each input exactly one output.
2. Identify if a relation represents a function given different representations (i.e., table, graph, mapping, story, patterns, equations, and ordered pairs).
3. Determine the independent and dependent variables in a functional relationship.

4.1a Class Activity: Introduction to Functions

1. Jason is spending the week fishing at the Springville Fish Hatchery. Each day he catches 3 fish for each hour he spends fishing. This relationship can be modeled with the equation $y = 3x$, where x = number of hours spent fishing and y = the number of fish caught.

    a. Complete the graph and table below for this relationship.

Number of hours spent fishing x	Number of fish caught y
3	
2	
1	
2	
0	
4	

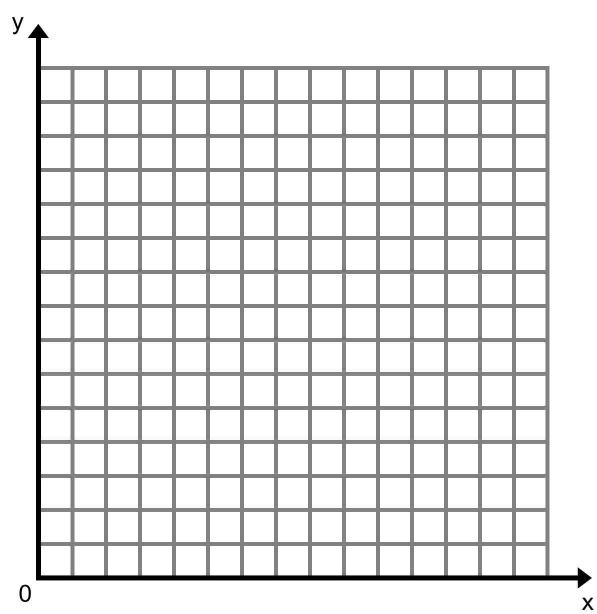


The situation above is an example of a **function**. We would say that *the number of fish caught is a* ***function*** *of the number of hours spent fishing.*

2. Sean is also spending the week fishing; however he is fishing in the Bear River. Each day he records how many hours he spends fishing and how many fish that he caught. The table of values below shows this relationship.

    a. Complete the graph for this relationship.

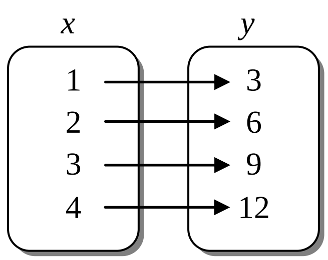
Number of hours spent fishing x	Number of fish caught y
1	4
0	0
2	5
3	1
3	8
5	5



This situation is an example of a relation that is **not a function**. The number of fish that Sean catches is **not** a function of the number of hours he spends fishing.

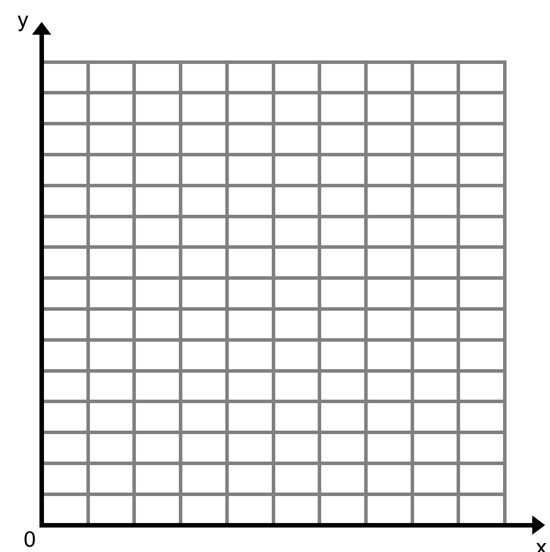
3. Compare and contrast the relationship for Jason's week spent fishing and Sean's week spent fishing. Make a conjecture (an educated guess) about what kind of relationship makes a function and what disqualifies a relation from being a function.

4. Vanessa is buying gumballs at Vincent's Drug Store. The mapping below shows the relationship between number of pennies, or *x*, she puts into the machine and the number of gumballs she gets out, or *y*.



   a. Complete the graph and table below for this relationship.

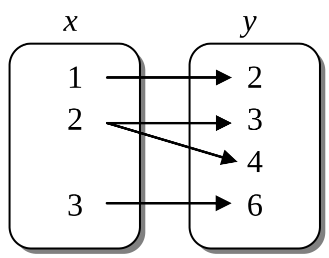
Number of pennies x	Number of gumballs y



   b. Write an equation that models this relationship.

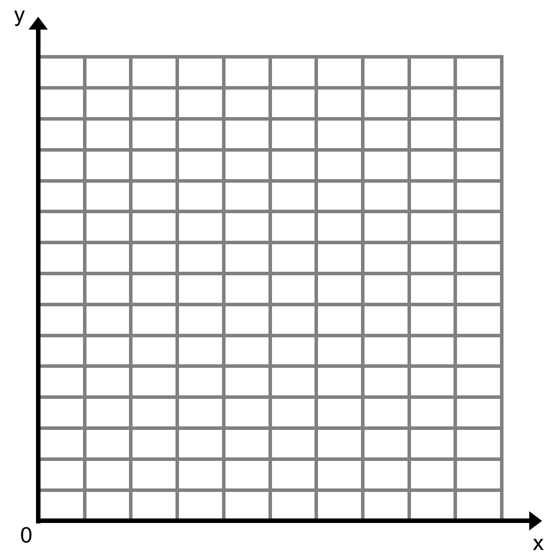
This is also an example of a **function.** We would say that *the number of gumballs received is a* ***function*** *of the number of pennies put in the machine.*

5. Kevin is across town at Marley's Drug Store. The mapping below relates the number of pennies he puts into the machine and how many gumballs he get outs.



a. Complete the graph and table below for this relationship.

Number of pennies x	Number of gumballs y

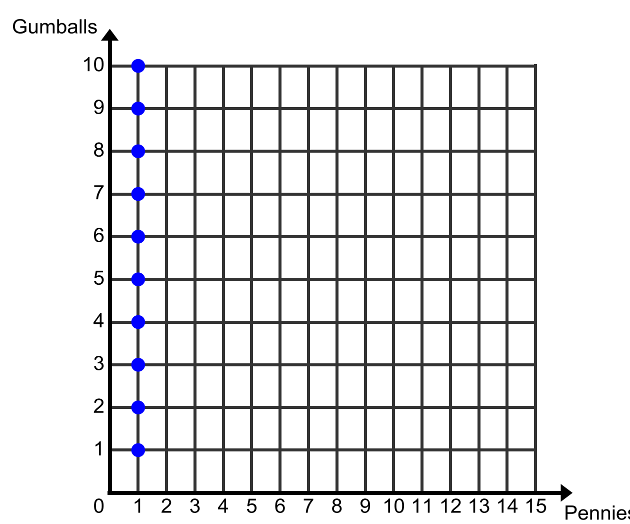


This situation is an example of a relation that is **not a function**.

6. Cody is at Ted's Drug Store. The graph below relates the number of pennies he puts into the machine on different occasions and how many gumballs he gets out.

a. Explain how this gumball machine works.

b. Do you think that this relation is an example of a function? Explain your answer.



7. Compare and contrast the relationship of the gumball machines at the different drugstores. If needed revise your conjecture about what kind of relationship makes a function and what disqualifies a relationship from being a function.

Below is a formal definition of a function. As you read it compare it to the conjecture you made about what makes a relation a function.

Given two variables, *x* and *y*, *y* is a function of *x* if there is a rule that determines one unique *y* value for a given *x* value.

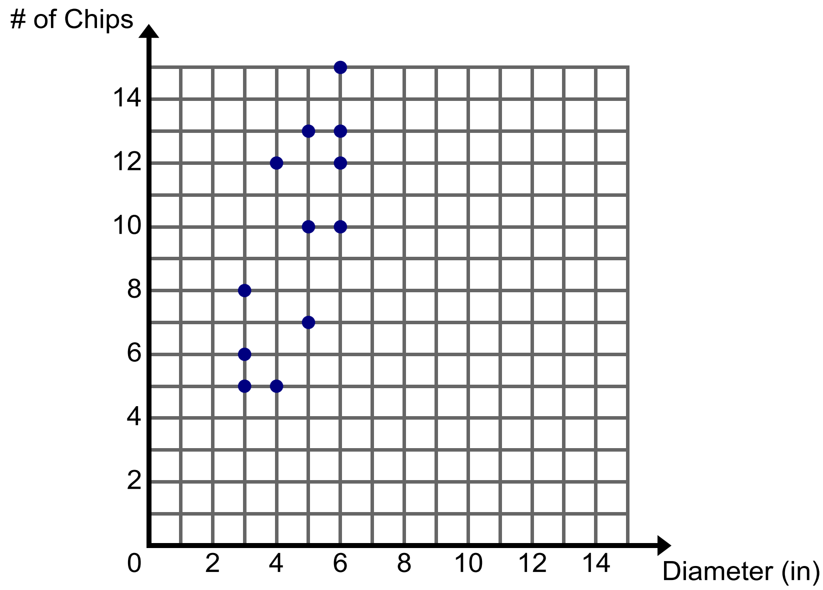
Refer back to the first two examples. Jason's pay is a function because each hour he works generates a unique amount of money made. In other words the amount of money made depends on the number of hours worked. Sean's pay is **not** a function because one day he worked 3 hours and made $45 and another day he worked 3 hours and made $50. There are two different *y* values assigned to the *x* value of 3 hours. The same is true when he works 4 hours. It is not possible to determine how much Sean will make based on the number of hours he works; therefore his pay is **not** a function of the number of hours he works.

Likewise, the gumball machine at Vincent's Drug Store represents a function because each penny inserted into the gumball machine generates a unique amount of gumballs. If you know how many pennies are inserted into the gumball machine at Vincent's, you can determine how many gumballs will come out. However, the gumball machine at Marley's Drug Store is **not** a function because there is not a unique number of gumballs generated based on the number of pennies you put in. At one point 2 pennies were inserted and 4 gumballs came out and at another point 2 pennies were inserted and 3 gumballs came out. You are unable to determine the number of gumballs that will come out based on how many pennies are put into the machine.

8. Explain in your own words why the gumball machine at Ted's Drug store is not a function. Be specific and give examples to support your reasoning.

4.1a Homework: Introduction to Functions

1. Betty's Bakery makes cookies in different sizes measured by the diameter of the cookie in inches. Curious about the quality of their cookies, Betty and her assistant randomly chose cookies of different sizes and counted the number of chocolate chips in each cookie. The graph below shows the size of each cookie and the number of chocolate chips it contains.

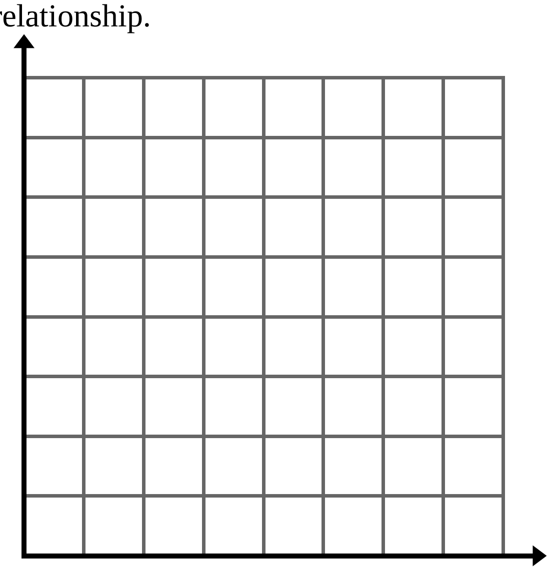


Diameter of Cookie (in) x	# of Chocolate Chips y

   a. Complete the table to the right of the graph.
   b. Is the situation above an example of a **function**? Why or why not?

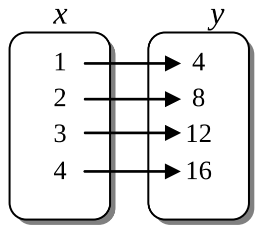
2. The number of tires y in the parking lot at Hank's Honda Dealership can be modeled by the equation $y = 4x$ where x represents the number of cars in the parking lot.
   a. Complete the graph and table below for this relationship.

Number of cars x	Number of tires y



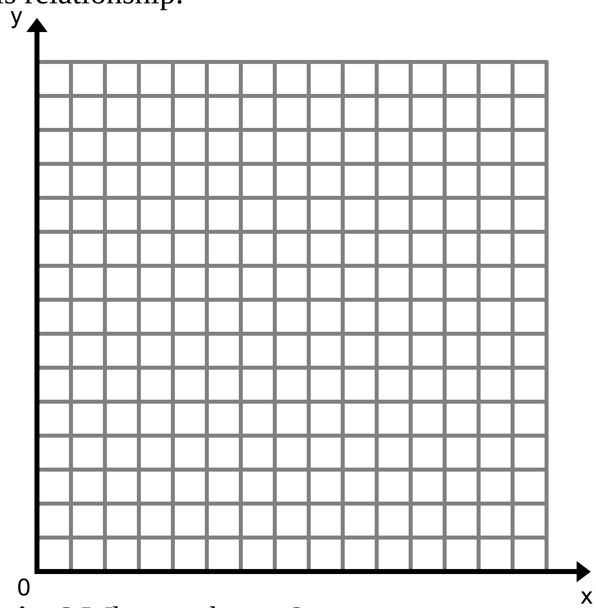
   b. Is the situation above an example of a **function**? Why or why not?

3. The cost for cars entering a scenic by-way toll road in Wyoming is given by the mapping below. In this relation y is the dollar amount to enter the by-way and x is the number of passengers in the car.



a. Complete the graph and table below for this relationship.

Number of passengers x	Amount per car y

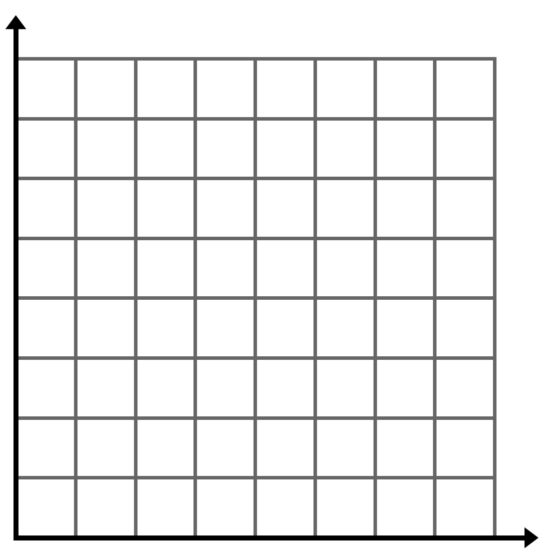


b. Is the situation above an example of a **function**? Why or why not?

4. The cost for cars entering a scenic by-way toll road in Utah is $5 regardless of the number of passengers in the car.

a. Complete the graph and table below for this relationship.

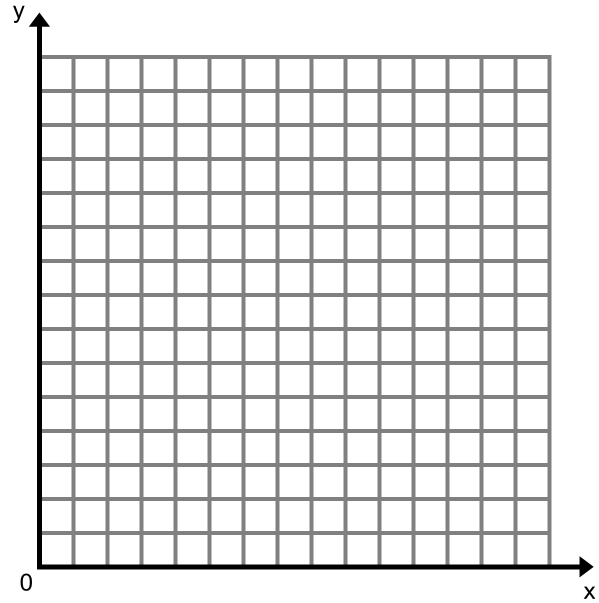
Number of passengers x	Amount per car y



b. Is the situation above an example of a **function**? Why or why not?

5. Create your own context or story that represents a relation that is a function.
   a. Story:

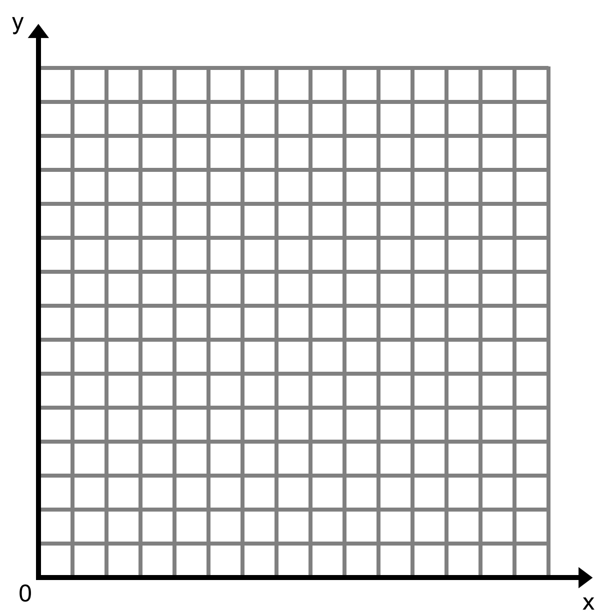
   b. Complete the graph and table below for this relationship.



   c. Explain why this relation is a function.

6. Create your own context or story that represents a relation that is **not** a function.
   a. Story:

   b. Complete the graph and table below for this relationship.

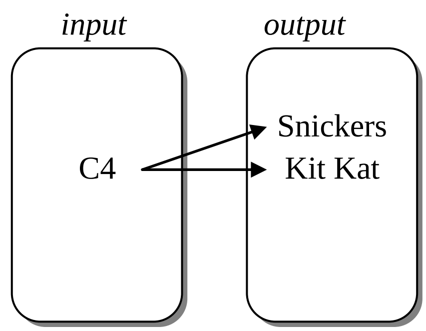


   c. Explain why this relation is not a function.

4.1b Class Activity: The Function Machine

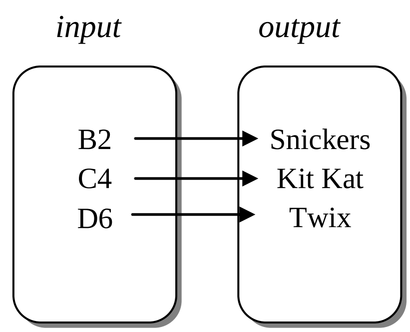
Another way to think about the *x* and *y* variables are as input and output values. To better understand how input and output values are related to a function investigate the following analogy.

The Candy Machine Analogy: When you buy candy from a vending machine, you push a button (your input) and out comes your candy (your output). Let's pretend that C4 corresponds to a Snickers bar. If you input C4, you would expect to get a Snickers bar as your output. If you entered C4 and sometimes the machine spits out a Snickers and other times it spits out a Kit Kat bar, you would say the machine is "not functioning" – one input (C4) corresponds to two different outputs (Snickers and Kit Kat).



Not a function!

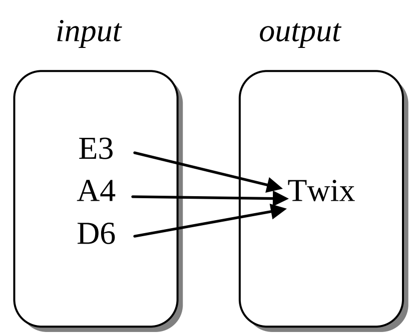
Let's look at what a diagram might look like for a machine that is "functioning" properly:



A Function!

In this situation, each input corresponds to exactly one output. The candy bar that comes out of the machine is dependent on the button you push. We call this variable the **dependent variable**. The button you push is the **independent variable**.

Let's look at one more scenario with the candy machine. There are times that different inputs will lead to the same output. In the case of the candy machines, companies often stock popular items in multiple locations in the machine. This can be represented by the following diagram:

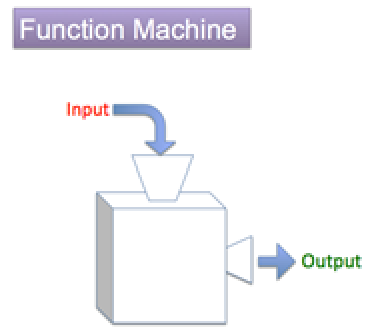


A Function!

Even though the different inputs correspond to the same output, our machine is still "functioning" properly. This still fits the special requirement of a function – each input corresponds to exactly one output.

In the candy machine each input value generated one unique output value.

We are going to continue the analogy of a function working as a machine by playing "Name that Function." You will be giving your teacher a number and she/he will do some operations to that number and give you a number back. You need to figure out what operations she/he is doing to that number. You can imagine a number going into a "function machine" and coming out a different number. Something is happening to that number in the machine and you are trying to figure out what is happening (what operations). Use these tables to help you keep track of the numbers.



Number you give the teacher	Number you get from the teacher

Function:

Number you give the teacher	Number you get from the teacher

Function:

Number you give the teacher	Number you get from the teacher

Function:

Number you give the teacher	Number you get from the teacher

Function:

Number you give the teacher	Number you get from the teacher

Function:

Number you give the teacher	Number you get from the teacher

Function:

Now try it on your own or with a partner. Write the function for each of the following relations. Use the words input and output in your function equation. Then try to write the function as an equation using x and y. The first one has been done for you.

1.		
Input	Function: $2 \cdot (input) + 1$ = y=2x+1	Output
8		17
0		1
-3		-5
2		5
1		3
-1		-1
7		15

2.		
Input	Function:___________________	Output
3		-2
0		-5
10		5
2		-7
5		0
1		-4
-4		-9

3.		
Input	Function: ___________________	Output
5		25
-2		-10
0		0
1		5
-4		-20
-9		-45
3		15

4.		
Input	Function: ___________________	Output
-3		-4
-2		-2
-1		0
0		2
1		4
2		6
3		8

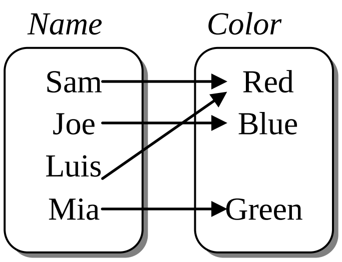
5.		
Input	Function: ___________________	Output
4		-2
-3		-16
0		-10
1		-8
7		4
9		8
5		0

6.		
Input	Function: ___________________	Output
2		3
5		-9
2		7
5		10
4		9
2		0
-1		-3

7. Were you able to find a function for number 6? If so, write it down. If not, explain why.

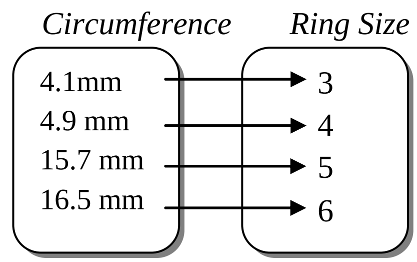
Functions can also be described by non-numeric relations. A mapping is a representation of a function that helps to better understand non-numeric relations. Study each non-numeric relation and its mapping below. Then decide if the relation represents a function. Explain your answer.

1. Student's name versus the color of their shirt.



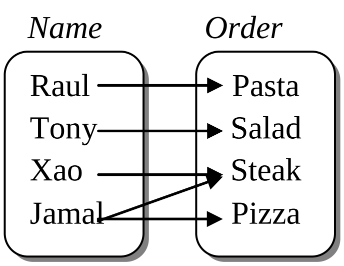
Function? Explain your answer.

2. Circumference of your finger versus your ring size



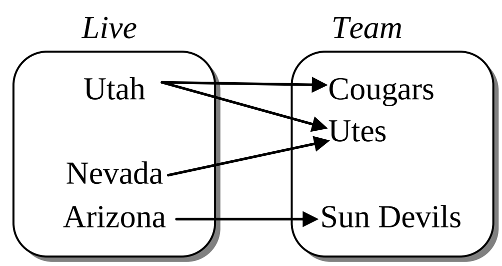
Function? Explain your answer.

3. Person's name versus what they order at a restaurant.



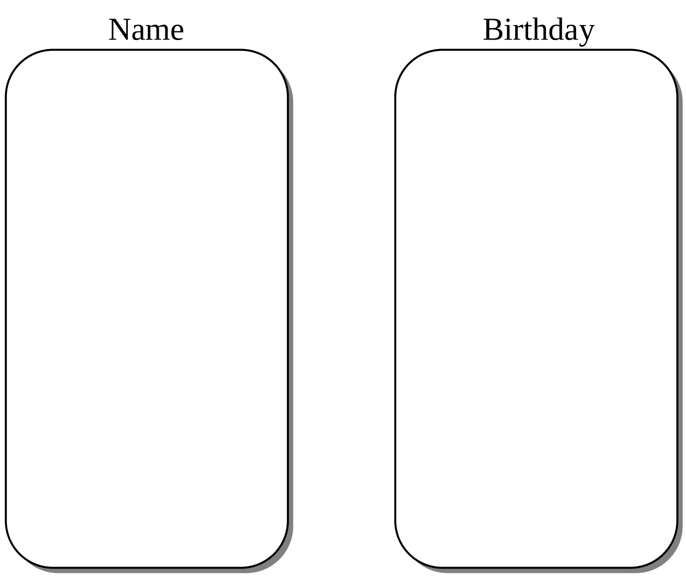
Function? Explain your answer.

4. Where a person lives versus the team they root for in college football.



Function? Explain your answer.

5. Make a mapping of the students' names (first and last) in your classroom vs. their birthday.



a. Is this relation a function? Justify your answer.

4.1b Homework: Function Machine

Directions: Write the function for each of the following relations.

1.		
Input	Function: ____________________	Output
5		2.5
-2		-4.5
0		-2.5
1		-1.5
-4		-6.5
-9		-11.5
3		0.5

2.		
Input	Function: ____________________	Output
2		1/2
-4		-1
0		0
1		¼
-9		-9/4
24		6
8		2

3.		
Input	Function: ____________________	Output
-4		-9
-3		-6
-2		-3
0		3
1		6
2		9
3		12

4.		
Input	Function: ____________________	Output
2		3
-4		-3
0		1
1		2
-9		-8
-17		-16
10		11

5.		
Input	Function: ____________________	Output
-4		16
-3		9
-2		4
-1		1
0		0
1		1
2		4

6.		
Input	Function: ____________________	Output
1		3
6		-8
1		7
5		10
6		8
2		0
-1		-3

7. Were you able to find a function for numbers 6? If so, write it down. If not, explain why.

Directions: Create your own function machines, fill in the values for each input and its corresponding output.

8.		
Input	Function:________________ =	Output

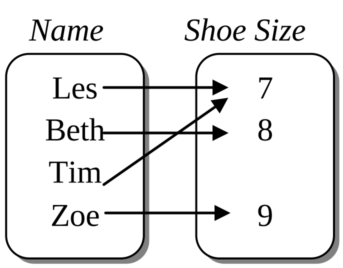
9.		
Input	Function:________________ =	Output

10. Create a machine that is **not** a function. Explain why your machine is "dysfunctional".

Input	Function:________________ =	Output

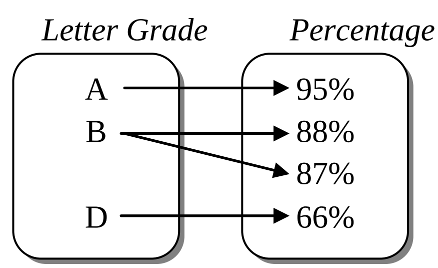
Directions: Study each non-numeric relation and its mapping below. Then decide if the relation represents a function. Explain your answer.

11. Student's name versus their shoe size.



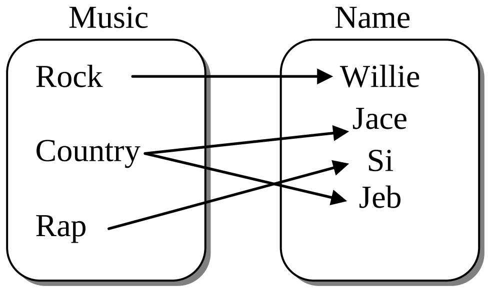
Function? Explain your answer.

12. A person's letter grade versus their percentage.



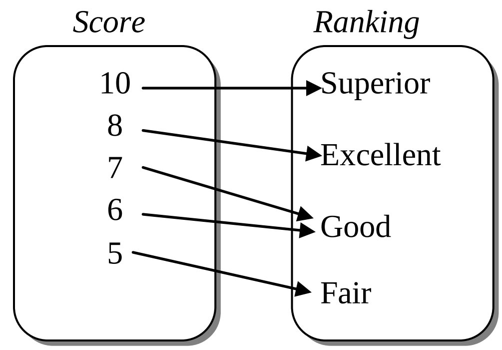
Function? Explain your answer.

13. Favorite type of music versus a person's name.



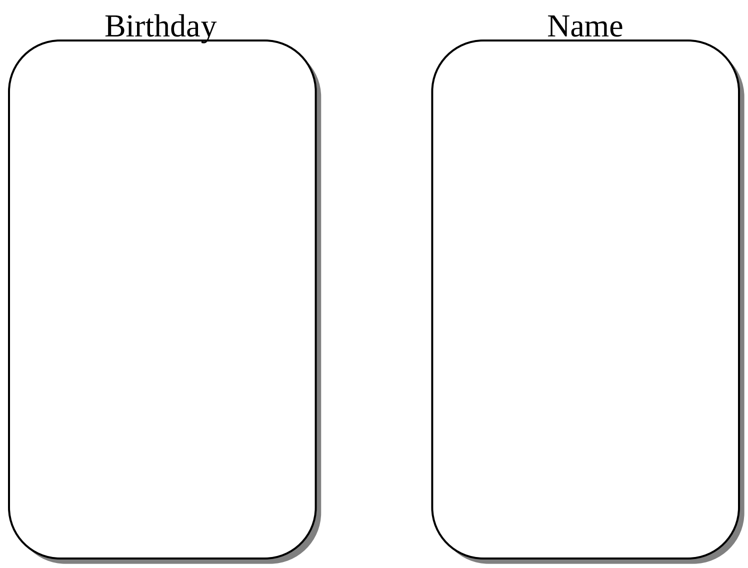
Function? Explain your answer.

14. A pianist's overall score in a music competition versus their ranking.



Function? Explain your answer.

15. Use the data gathered from number in the class activity above to make a mapping of the relation, a person's birthday versus name in your classroom. Is this relation a function? Use their first and last name. Justify your answer.



4.1c Class Activity: Representations of a Function

Write down the definition of a function in your own words in the box below.

Directions: There are many ways to represent a relation or function. Investigate the different representations of a relation given below and determine if the relationship is a function or not.

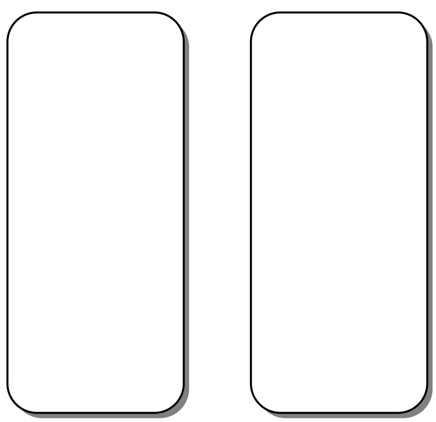
1. Story: A candle is 27 centimeters high and burns 3 centimeters per hour. An equation that models this relation is $c = 27 - 3h$ where c is the height of the candle in centimeters and h is the number of hours.
    a. Express this relation as a table, mapping, and graph.

Table

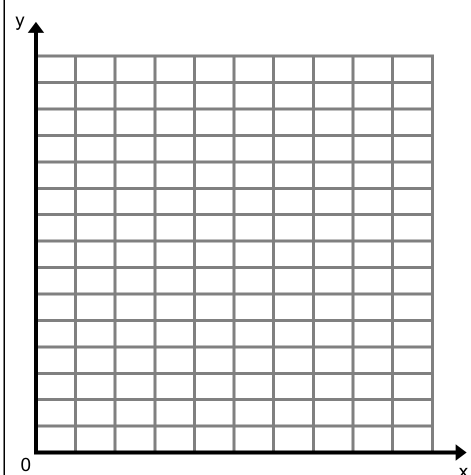
Time (hours) h	Height (cm) c

Mapping

Hours *Centimeters*

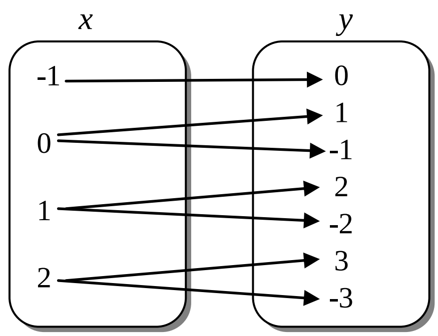


Graph



    b. Is this relation a function? Explain how you know.

2. Mapping:

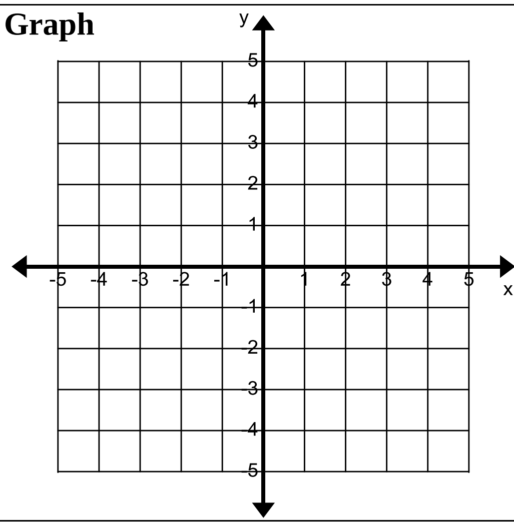


a. Express this relation as a table and graph.

Table

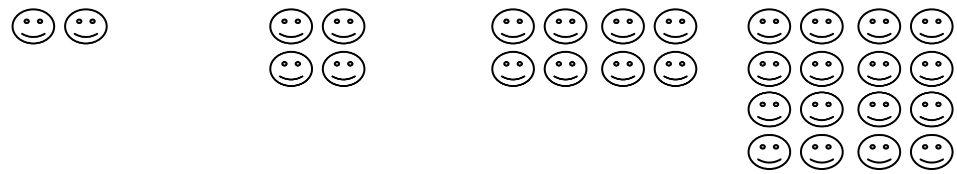
x	*y*

Graph



b. Is this relation a function? Explain how you know.

3. Pattern:



Stage 1 Stage 2 Stage 3 Stage 4

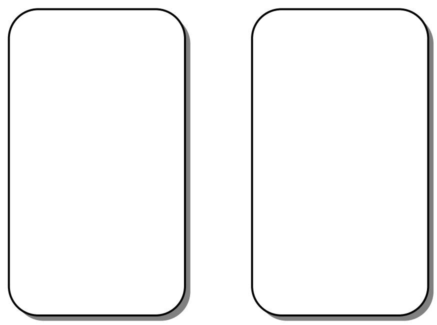
a. Express this relation as a table, mapping, and graph.

Table

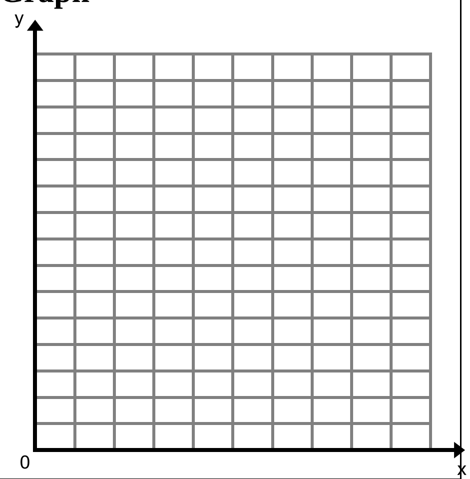
Stage number	Number of Smiles

Mapping

Stage *Smiles*

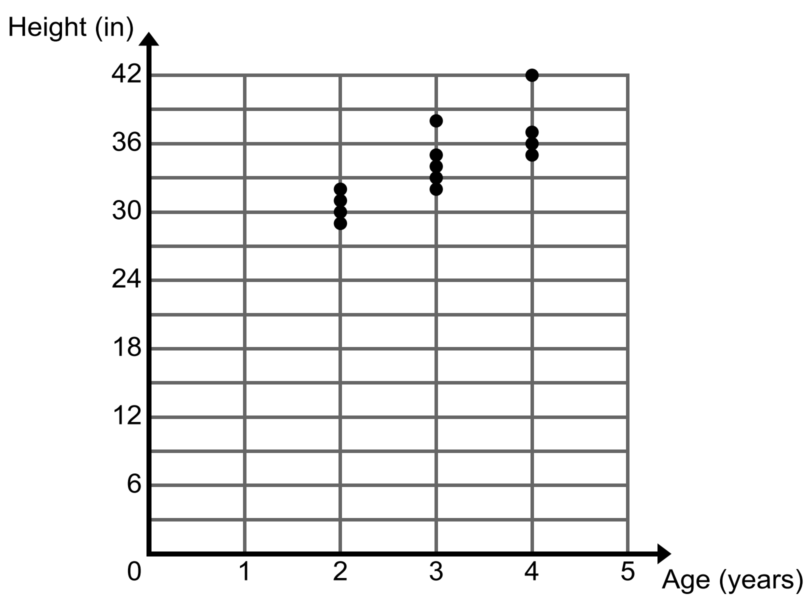


Graph



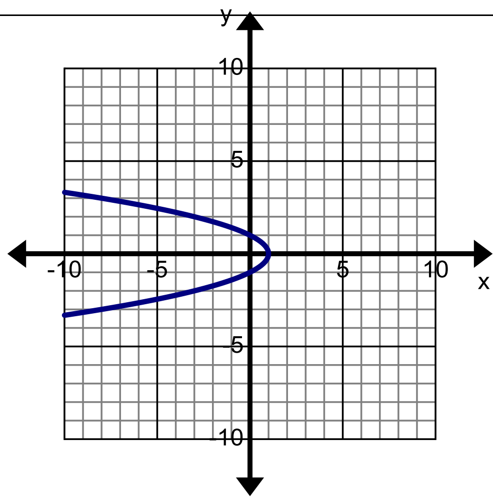
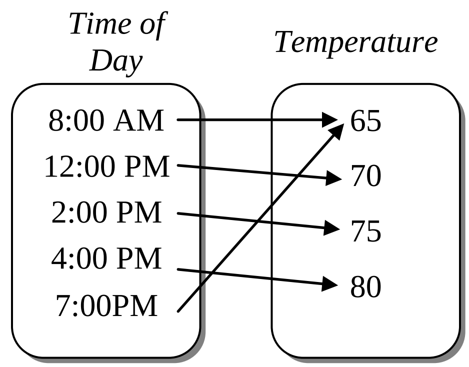
b. Is this relation a function? Explain how you know.

4. Graph: Discovery Place Preschool gathered data on the age of each student (in years) vs. the child's height (in inches). The graph below displays the data they gathered.



a. Is this relation a function? Explain how you know.

Directions: Determine if each relation or situation is a function. Justify your choice.

5. {(30, 2), (45, 3), (32, 1.5), (30, 4), (41, 3.4)}	6. A student's name versus their student ID number.
7. 	8. 

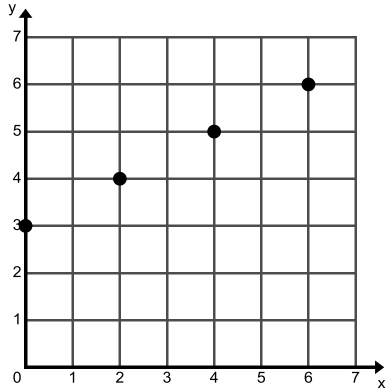
4.1c Homework: Representations of a Function

Directions: Determine if each relation or situation could be represented by a function. Justify your choice.

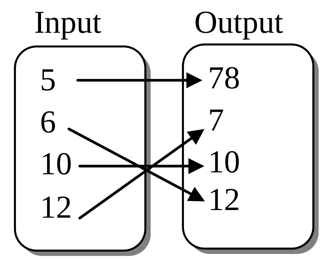
1.

Input	**Output**
0.2	1.5
0.4	1.25
0.6	1.5
0.8	1.25

2.



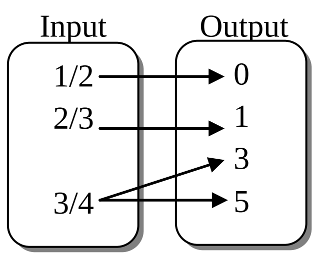
3.



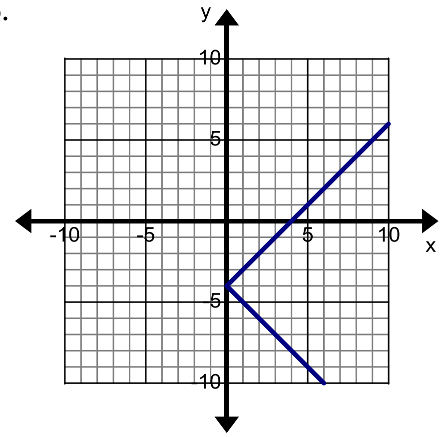
4.

Input	**Output**
25	14
30	13
30	12
35	11

5.



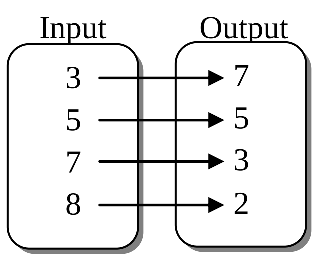
6.



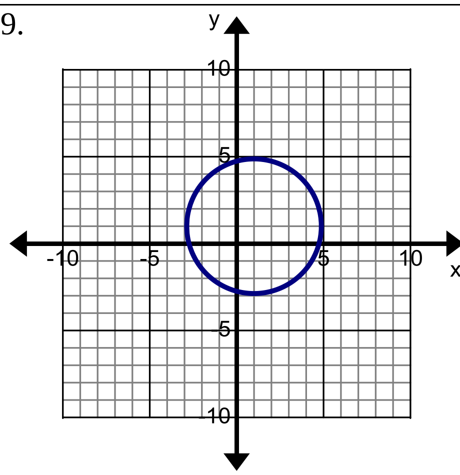
7.

Number of People	1	3	4	7
Cost	4	8	10	16

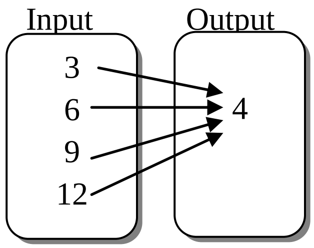
8.



9.



10.



11.

$$y = \frac{1}{3}x + 4$$

12.

$$y = -5$$

13. You know your cousin lives at the zip code 12345 so you type it in Google to find your cousin's full address.

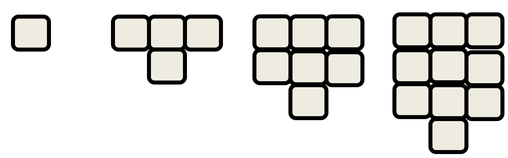
14. You know your cousin's cellular phone number is (123) 456-7890 so you dial that number to call him.

15. You call the post office and ask for the zip code for the city of Nephi, UT

16.

x Pounds	1.5	2.3	3.1	4.2
y dollars	\$7.80	\$11.96	\$16.12	\$21.82

17.



Stage 1 Stage 2 Stage 3 Stage 4

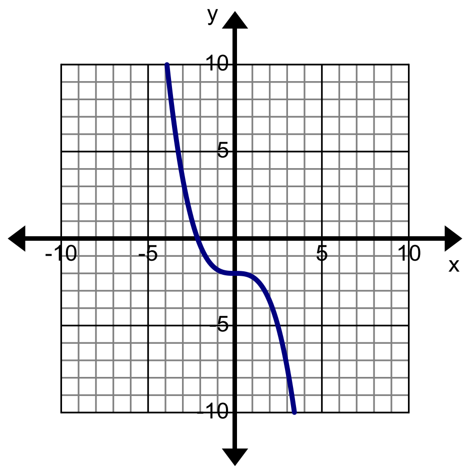
18. {(2, 3), (5, -1), (3, 4), (5, 2)}

19. {(-1, 0), (1, 2), (1, 4), (5, 2)}

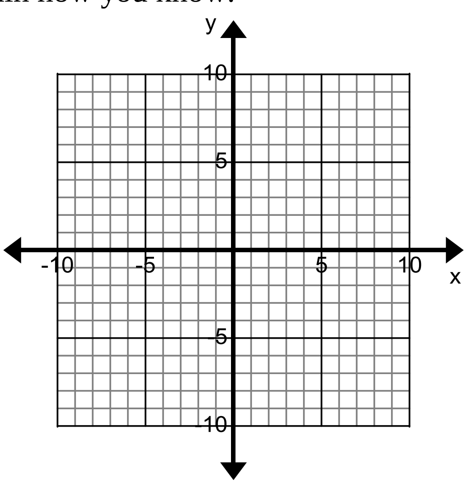
20.

Stage 1 Stage 2 Stage 3 Stage 4 Stage 5

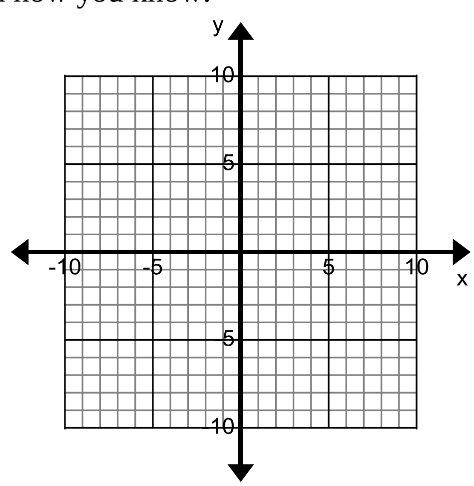
21.



22. Draw a graph of a relation that is a function. Explain how you know.



23. Draw a graph of a relation that is **not** a function. Explain how you know.



24. Make a mapping of a relation that is a function. Explain how you know.

25. Creat a set of ordered pairs that do **not** represent a function. Explain how you know.

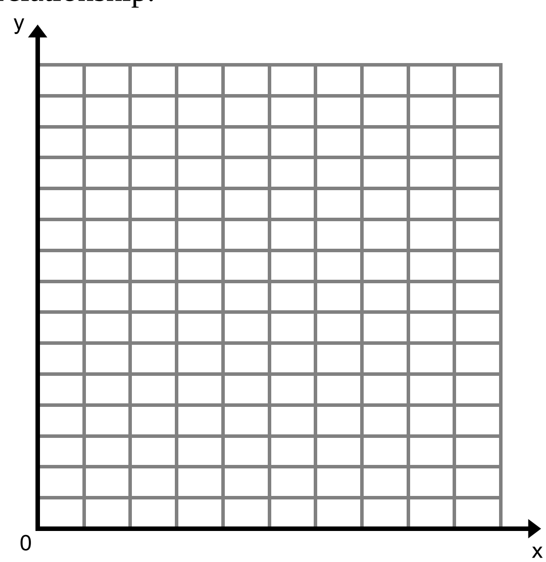
4.1d Class Activity: Independent and Dependent Variables

1. Paradise Valley Orchards has the banner shown hanging from their store window. Sally is trying to determine how much she will spend depending on how many bushels of apples she purchases.

   a. Write an equation that gives the amount Sally will spend y depending on how many bushels of apples x she purchases.

   b. Complete the graph and table below for this relationship.

Number of Bushels x	Amount Spent (dollars) y



We know from the previous lessons, that the relationship between number of bushels purchased and amount spent is an example of a function. The equation above gives us a rule for how to determine the amount of money spent based on the number of bushels purchased.

In a functional relationship represented with an equation, the **independent variable** represents the input or x-value of the function and the **dependent variable** represents the output or y-value of the function. In a function, the **dependent variable** is determined by or depends on the **independent variable**. In our example above the **independent variable** is the number of bushels purchased and the **dependent variable** is the amount of money spent. The amount of money one spends **depends** on the number of bushels one purchases. Another way to say this is that the amount of money spent is a function of the number of bushels purchased.

If we think of our input machine, we are inputing the number of bushels purchased and the machine takes that number and multiplies it by 15 to give us our output which is the amount of money we will spend.

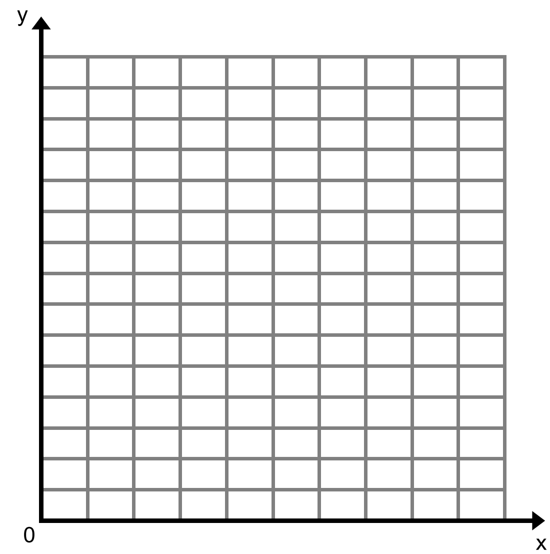
2. Miguel is taking a road trip and is driving at a constant speed of 65 mph. He is trying to determine how many miles he can drive based on how many hours he drives.

   a. Identify the **independent variable** in this situation: ________________________

   b. Identify the **dependent variable** in this situation: ________________________

   c. Complete the graph and table below for this relationship. Make sure you label the columns and axes in your table and graph.

x	*y*



   d. Write an equation that represents this situation: ______________________________

   e. In this situation ___________________________ is a function of _________________________.

Directions: Each of the following situations represents a function. Put an I above the independent variable and a D above the dependent variable.

3. The drama club is selling tickets to the Fall Ball. The more tickets that they sell increases the amount of money they can spend on decorations.

4. Generally, the average price of going to the movies has steadily increased over time.

5. In warm climates the average amount of electricity used rises as the daily average temperature increases and falls as the daily average temperature decreases.

6. The number of calories you burn increases as the number of minutes that you walk increases.

7. The air pressure inside a tire increases with the temperature.

8. As the amount of rain decreases, so does the water level of the river.

9. There are approximately 30 pickles in a jar of dill pickles. The total number of jars of pickles that a factory can produce depends on the number of pickles they receive.

10. Linda buys a case of pork and beans. The weight of the box increases as the number of cans increases.

11. Susan has 5 minutes to study for a test; she read (scanned) 10 pages in 5 minutes.

12. Chris has 30 pages left in his book; it takes him 45 minutes to finish the book.

4.1d Homework: Independent and Dependent Variables

1. Shari is filling up her gas tank. She wants to know how much it will cost to put gas in her car. The sign below shows the cost for gas at Grizzly's Gas-n-Go.

PRICE OF GAS

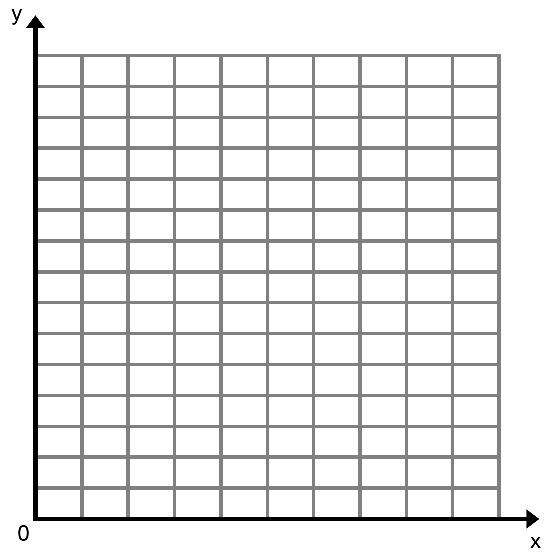
$3.25/GALLON

   a. Identify the **independent variable** in this situation: ________________________

   b. Identify the **dependent variable** in this situation: ________________________

   c. Complete the graph and table below for this relationship. Make sure you label the columns and axes in your table and graph.

x	*y*



   d. Write an equation that represents this situation: ______________________________

   e. In this situation ___________________________ is a function of _________________________.

2. Peter is the event planner for a relay race taking place in Park City, UT. He needs to determine how many bottles of water to have ready at the finish line of the race so that each participant in the race receives a bottle of water. There are 4 people on each a team.

   a. Identify the **independent variable** in this situation: ________________________

   b. Identify the **dependent variable** in this situation: _________________________

   c. Write an equation that represents this situation: ______________________________

   d. In this situation ___________________________ is a function of _________________________.

Directions: Each of the following situations represents a function. Put an I above the independent variable and a D above the dependent variable.

3. As the size of your family increases so does the cost of groceries.

4. The value of your car decreases with age.

5. The speed of a sprinter vs. the time it takes to run a race.

6. The distance you can drive vs. the amount of gas in the tank.

7. A child's wading pool is being inflated. The pool's size increases at a rate of 2 cubic feet per minute.

8. The total number of laps run depends on the length of each workout.

9. A tree grows 15 feet in 10 years.

10. There are 5 inches of water in a bucket after a 2 ½ hour rain storm.

13. Jenny has 30 coins she has collected over 6 years.

14. Sally's track coach wants to know Sally's speed in miles per hour. She can run 4 miles in 2/3 of an hour.

15. Whitney is training for a half marathon and is scheduled to go for a 9 mile run for her training, it takes her 80 minutes to complete the run.

16. Write your own relationship that contains an independent and dependent variable.

4.1e Self-Assessment: Section 4.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Deep Understanding, Skill Mastery
1. Understand that a function is a rule that assigns to each input exactly one output.			
2. Identify if a relation represents a function given different representations (i.e., table, graph, mapping, story, patterns, equations, and ordered pairs).			
3. Determine the independent and dependent variables in a functional relationship.			

Section 4.2: Explore Linear and Nonlinear Functions

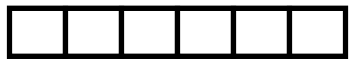
Section Overview:
This section focuses on the characteristics that separate linear from nonlinear functions. Students will analyze the different representations of a function (graph, table, equation, and context) to determine whether or not the representations suggest a linear relationship between the two variables. Students will solidify their understanding of how a linear function grows (changes). Lastly, students will compare properties of linear functions each represented in a different way.

Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Distinguish between linear and nonlinear functions given a context, table, graph, or equation.
2. Understand how a linear function grows (changes).
3. Match the representations (table, graph, equation, and context) of linear and nonlinear situations.
4. Compare properties of linear functions (rates of change and intercepts) and use this information to solve problems.

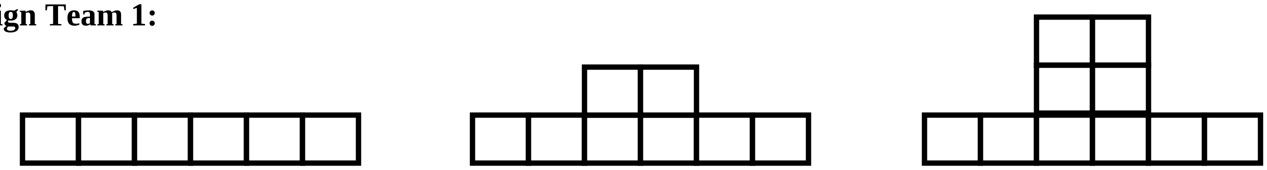
4.2a Class Activity: Display Designs

Complete Foods, a local grocery store, has hired three different companies to come up with a display for food items that are on sale each week. They currently have a display that is 6 boxes wide as shown below.



They would like the center part of the display to be taller than the outside pieces of the display to showcase their “mega deal of the week”. The following are the designs that two different companies submitted to Complete Foods, using the current display as their starting point.

Design Team 1:



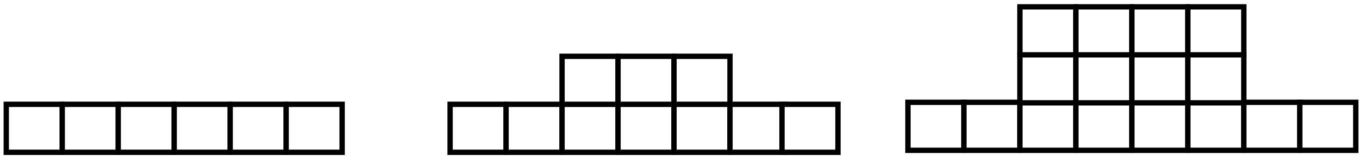
Current Display (Stage 1) **Stage 2** **Stage 3**

Stage 4

1. Draw Stage 4 of this design. Describe how you went about drawing stage 4.

2. Can the relationship between stage number and number of blocks in a stage in this pattern be modeled by a linear function? Provide at least **2** pieces of evidence to support your answer.

Design Team 2:



Current Display (Stage 1) **Stage 2** **Stage 3**

Stage 4

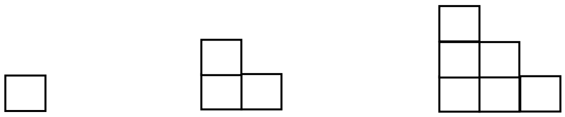
3. Draw Stage 4 of this design. Describe how you went about drawing stage 4.

4. Can the relationship between stage number and number of blocks in a stage in this pattern be modeled by a linear function? Provide at least **2** pieces of evidence to support your answer.

4.2a Homework: More Patterns – Are They Linear?

Directions: For each of the following patterns, draw the next stage and determine whether relationship between the stage number and the number of blocks in a stage can be represented by a linear function. Justify your answer.

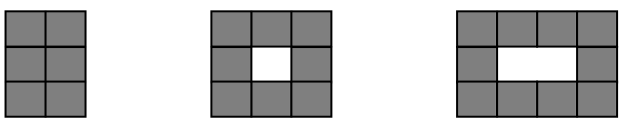
1.



Stage 1 Stage 2 Stage 3 Stage 4

a. Is this pattern linear? ________________________
b. Justification:

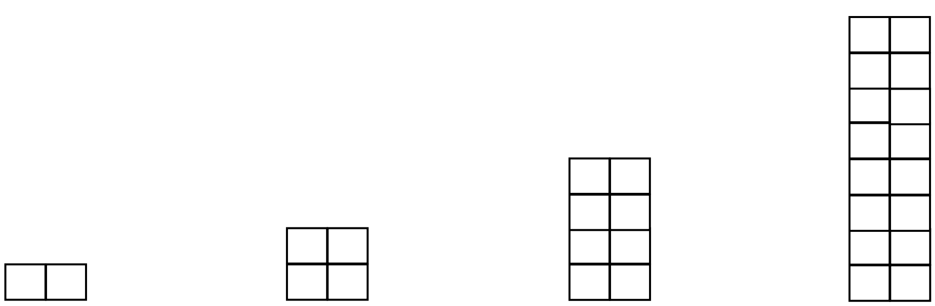
2. Consider the gray tiles only



Stage 1 Stage 2 Stage 3 Stage 4

a. Is this pattern linear? ________________________
b. Justification:

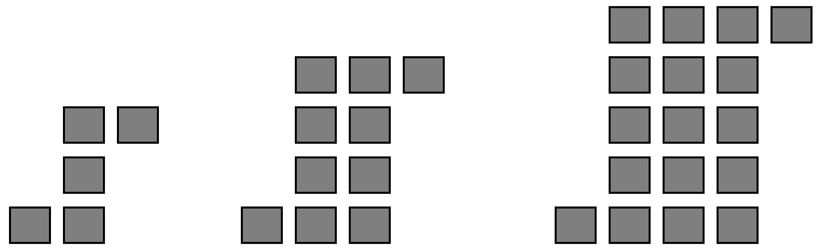
3.



Stage 1 Stage 2 Stage 3 Stage 4 Stage 5

a. Is this pattern linear? ________________________
b. Justification:

4.



Stage 1 Stage 2 Stage 3 Stage 4

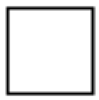
a. Is this pattern linear? ______________________
b. Justification:

5. Make up your own pattern that is **not** linear. Prove that your pattern is not linear with at least **2** pieces of evidence.

4.2b Classwork: Linear and NonLinear Functions in Context

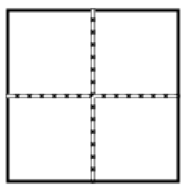
1. Consider the **area** of a square as a function of the side length of the square.
   a. Draw pictures to represent these squares. The first two have been drawn for you.

A = 1



Side length = 1

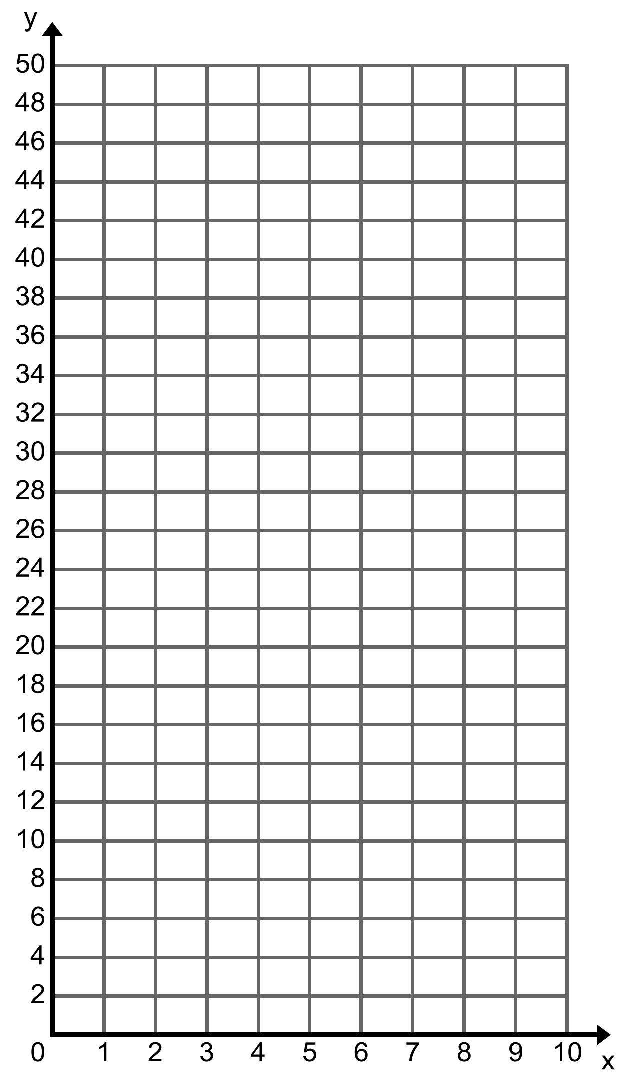
A = 4



Side length = 2

   b. Complete the graph and table for this function.

side length	area
1	
2	
3	
4	
5	



   c. What is the dependent variable? The independent variable?

   d. Write an equation to model *A* as a function of *s*.

   e. Does this graph pass through the point (8, 64)? Explain how you know.

   f. What does the point (8, 64) represent in this context?

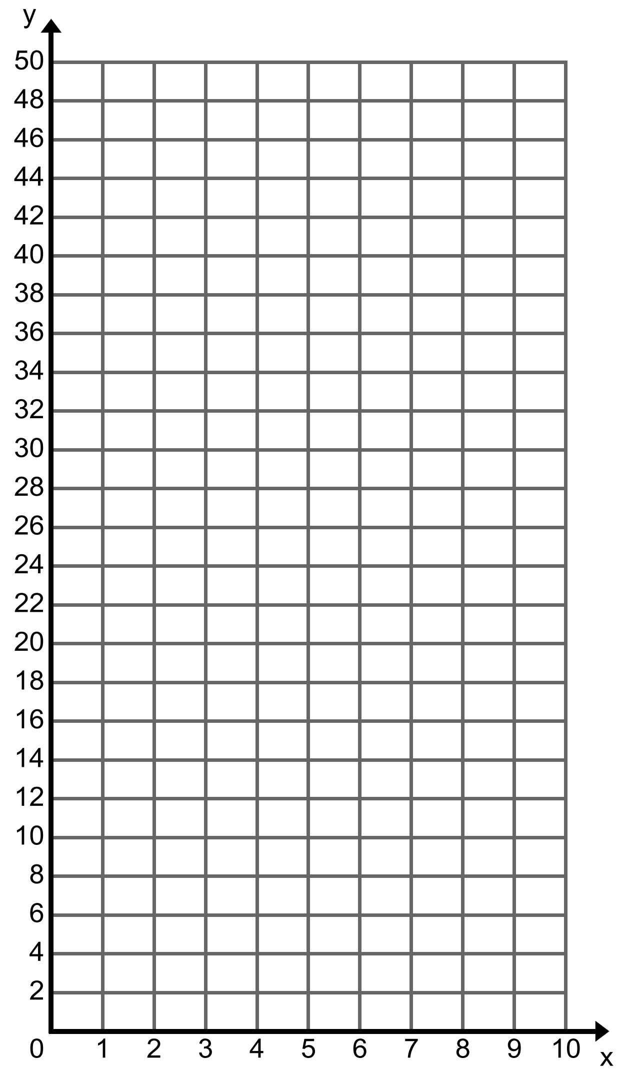
   g. List three more ordered pairs that this graph passes through.

   h. Is this function linear? Explain or show on the graph, table, and equation why or why not?

2. Consider the **perimeter** of a square as a function of the side length of the square.

a. Complete the graph and table for this function.

side length	perimeter
1	
2	
3	
4	
5	



a. What is the dependent variable? The independent variable?

b. Write an equation to model *P* as a function of *s*.

c. Find another ordered pair that the graph passes through.

d. What does the point from c) represent in this context?

e. Is this function linear? Explain or show on the graph, table, and equation why or why not?

4.2b Homework: Linear and NonLinear Functions in Context

1. The following tables show the distance traveled by three different cars over five seconds.

Car 1	
Time (s)	Distance (ft.)
1	4
2	7
3	10
4	13
5	16

Car 2	
Time (s)	Distance (ft.)
1	2
2	5
3	10
4	17
5	26

Car 3	
Time (s)	Distance (ft.)
1	3
2	5
3	9
4	17
5	33

   a. Consider time vs. distance traveled. Which of the tables of data can be modeled by a linear function? Which ones cannot be modeled by a linear function? Justify your answer.

   b. For any of the data sets that can be modeled by a linear function, write a function that models the distance traveled D as a function of time t.

   c. What is the dependent variable in this situation? The independent variable?

   d. Which car is traveling fastest? Justify your answer.

2. Hermione argues that the table below represents a linear function. Is she correct? How do you know?

x	2	4	8	16
y	1	3	5	7

3. Emily's little brother painted on her math homework. She knows the data in each of the tables below represents a linear function. Help Emily determine what number is hidden behind the blob of paint.

   a.

x	10	20	30	40
y	8	13		23

   b.

x	–2	0	2	3
y	–5		7	10

   c.

x	0	1		6
y	0	3	9	18

4.2c Class Activity: The Handshake Problem

On Tamara’s first day of math class, her teacher asked the students to shake hands with everyone in the room to introduce themselves. There are 26 students total in the math class. Can you determine the number of handshakes that took place in Tamara’s math class on the first day of class? Can the number of students vs. the number of handshakes exchanged be modeled by a linear function? Justify your answer.

4.2c Homework: Linear and NonLinear Situations

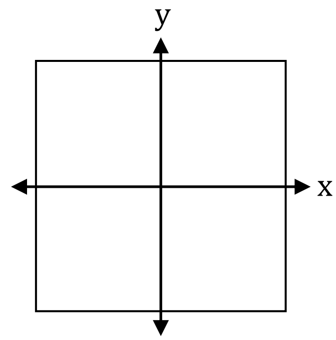
Directions: Choose **3** of the following situations. Determine whether the situations you choose can be modeled by a linear function or not. Provide evidence to support your claim. Show your work in the space below.

1. Mr. Cortez drove at a constant rate for 5 hours. At the end of 2 hours he had driven 90 miles. After 5 hours, he had driven 225 miles. Can time vs. distance driven be modeled by a linear function? Provide evidence to support your claim.
2. Round 1 of a tennis tournament starts with 64 players. After each round, half the players have lost and are eliminated from the tournament. Therefore, in round 2 there are 32 players, in round 3 there are 16 players and so on. Can round number vs. number of players be modeled by a linear function? Provide evidence to support your claim.
3. A rock is dropped from a cliff that is 200 feet above the ground. The table below represents the height of the rock (in feet) with respect to time (in seconds). Can time vs. height be modeled by a linear function? Provide evidence to support your claim.

Time (s)	**Height (ft.)**
0	200
1	184
2	136
3	56

4. A student comes to school with the flu and infects three other students within an hour before going home. Each newly infected student passes the virus to three new students in the next hour. This pattern continues until all students in the school are infected with the virus.
5. A piece of paper is cut into two equal sections. Each new piece is cut into two additional pieces of equal size. This pattern continues until it is no longer possible to cut the paper any more.

4.2d Classwork: Comparing Linear and NonLinear Equations



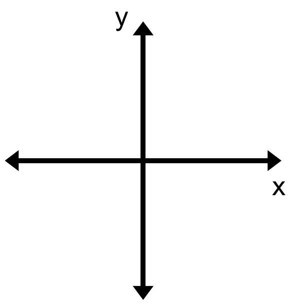
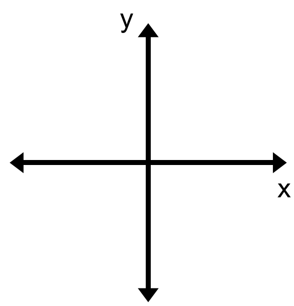
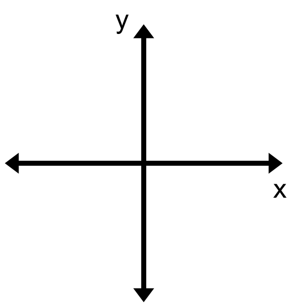
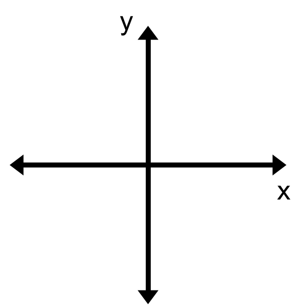
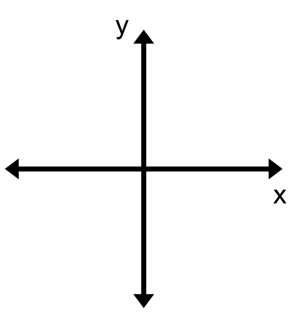
1. We know the graph of $y = mx + b$. Sketch the general appearance.
   a. What do m and b represent?

   b. What makes the graph linear?

2. Predict which of the following five graphs will be linear.

3. What other predictions can you make about what the graphs will look like?

4. Enter the equations into the graphing calculator and sketch the graphs below.

$y = \|x\|$ 	$y = x^2$ 
$y = \frac{1}{x}$ 	$y = \sqrt{x}$ 
$y = 2^x$ 	

5. Graph the equation $y = 2x^2 + 5$. How does this graph compare to the graph of $y = x^2$ above?

6. What do you think the graph of $y = \frac{1}{4}|x - 3| - 2$ will look like? Check your prediction by graphing.

7. Complete the table of values for the equations you graphed above. Using the table of values and the graphs from above, discuss why the equation makes the graph look like it does.

y = x	
x	***y***
-2	
-1	
0	
1	
2	

y = |x|	
x	***y***
-2	
-1	
0	
1	
2	

$y = x^2$	
x	***y***
-2	
-1	
0	
1	
2	

$y = \frac{1}{x}$	
x	***y***
-2	
-1	
0	
1	
2	

$y = \sqrt{x}$	
x	***y***
0	
1	
2	
3	
4	

$y = 2^x$	
x	***y***
0	
1	
2	
3	
4	

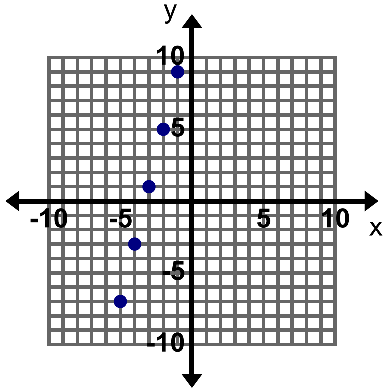
8. Describe the basic structure of an equation of a **linear function**. Think about the different forms a linear equation might take. Provide examples of the different forms.

9. Describe attributes you see in the equations of **nonlinear functions**. Provide examples.

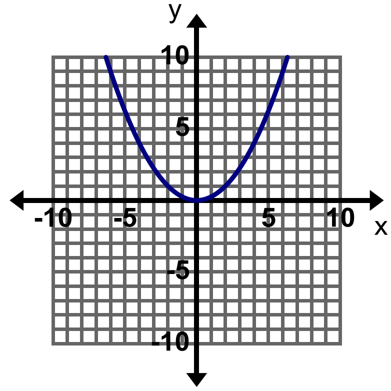
4.2d Homework: Representations of Linear and Nonlinear Functions

1. Circle the letter next to the graph if it represents a linear function.

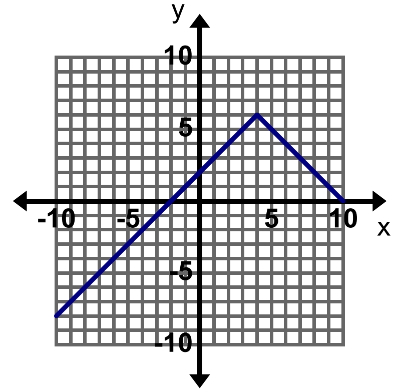
A



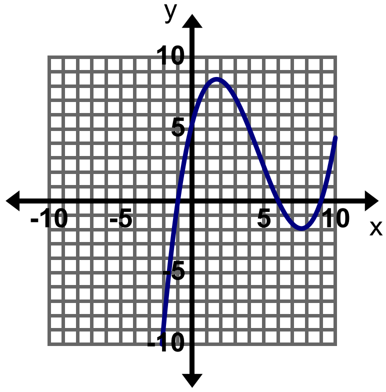
B



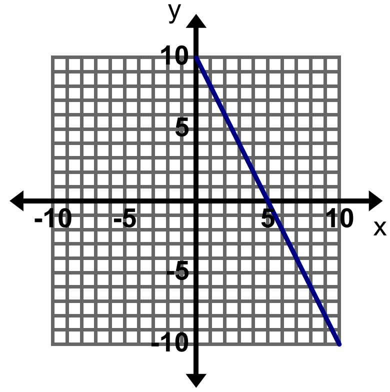
C



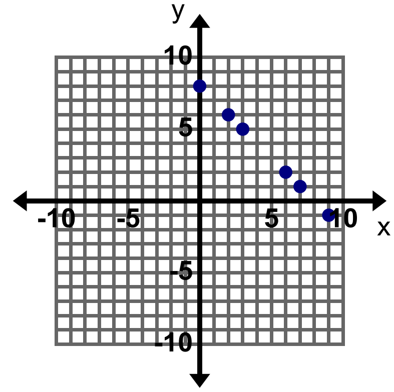
D



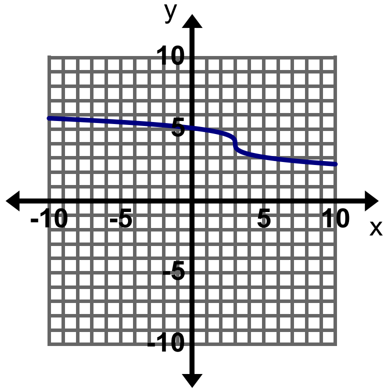
E



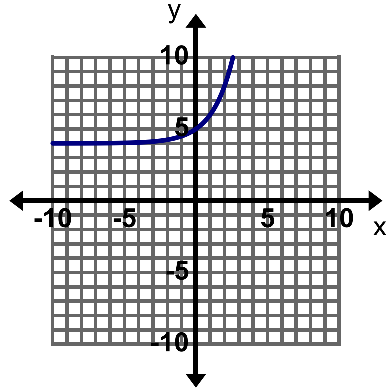
F



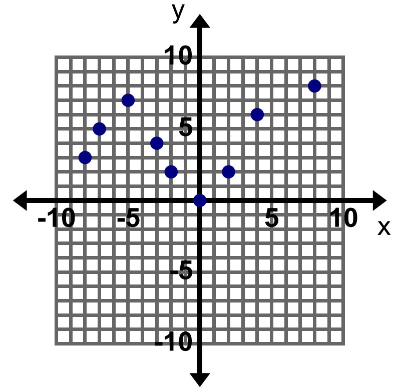
G



H



I



Bonus: Do any of the graphs of nonlinear functions have a shape similar to the ones studied in class? Make a prediction about the basic structure of the equations of these functions.

2. Circle the letter next to the table if the data represents a linear function.

A

x	*y*
0	-5
1	0
2	5
3	10

B

x	*y*
0	15
1	12.5
2	10
3	7.5

C

x	*y*
0	4
1	8
2	16
3	32

D

x	*y*
0	98
1	98
2	98
3	98

E

x	*y*
0.2	0.2
0.4	0.4
0.6	0.6
0.8	0.8

F

x	*y*
1	2
3	4
6	6
10	8

G

x	*y*
2.1	4
2.2	5
2.3	6
2.4	7

H

x	*y*
4	0
8	1
12	0
16	-1

I

x	*y*
3	20
6	24
12	32
24	48

J

x	*y*
10	-20
30	-40
50	-60
70	80

K

x	*y*
5	0
10	-1
15	-2
20	-3

L

x	*y*
15.1	4.2
16.7	12.2
18.3	20.2
19.9	28.2

3. Circle the letter next to each equation if it represents a linear function.

A	$2x + 4y = 16$	**B**	$y = \|2x\| + 5$	**C**	$y = x^2 + 5$	**D**	$y = 5 \cdot 3^x$
E	$y = \frac{4}{x} + 3$	**F**	$y = \frac{x}{4} + 3$	**G**	$y = \sqrt{4x}$	**H**	$x^2 + y^2 = 25$
I	$xy = 24$	**J**	$x + y = 6$	**K**	$y = -\frac{2}{3}x$	**L**	$y = 8$

Bonus: Can you predict the basic shape of any of the graphs of the nonlinear equations in #3?

4.2e Classwork: Representations of Functions
Matching Activity: Match the following representations together. Each representation will have a
1) a story,
2) an equation,
3) a table of values, and
4) a graph.

After you have matched the representations, **label** the axes of the graphs on the graph cards, **answer** the questions asked in the word problems on the story cards, and **identify** the dependent and independent variable in each story.

Story	Equation	Table	Graph
DD			
Z			
AA			
FF			
EE			
Y			
BB			
CC			

A

x	0	3	6	9	12	15
y	6	7	8	9	10	11

B

x	0	1	2	3	4	5
y	6	100	162	192	190	156

C

x	0	4	8	12	16	20
y	0	200	200	200	224	248

D

x	0	1	2	3	4	5
y	6	12	24	48	96	192

E

x	0	5	10	15	20	25
y	6	7	8	9	10	11

F

x	0	1	2	3	4	5
y	200	196	192	188	184	180

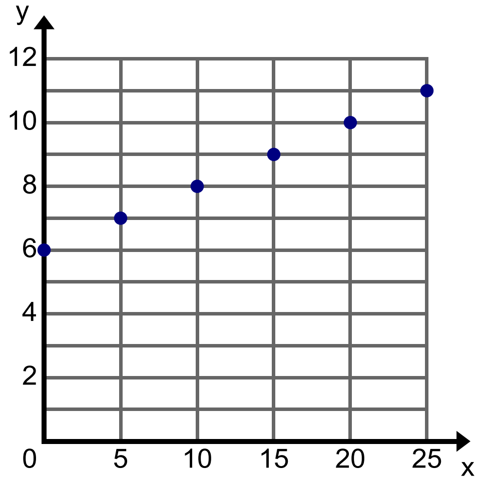
G

x	0	1	2	3	4	5
y	6	206	406	606	806	1006

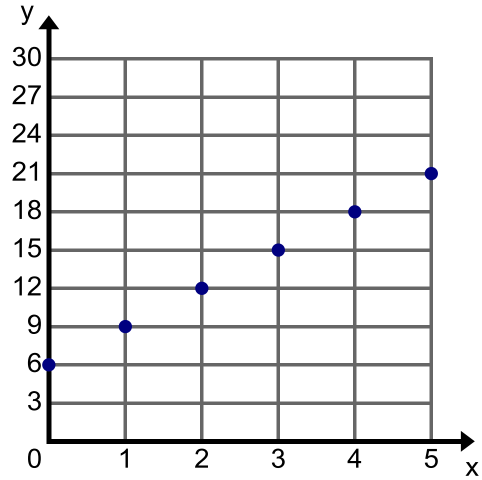
H

x	0	1	2	3	4	5
y	6	9	12	15	18	21

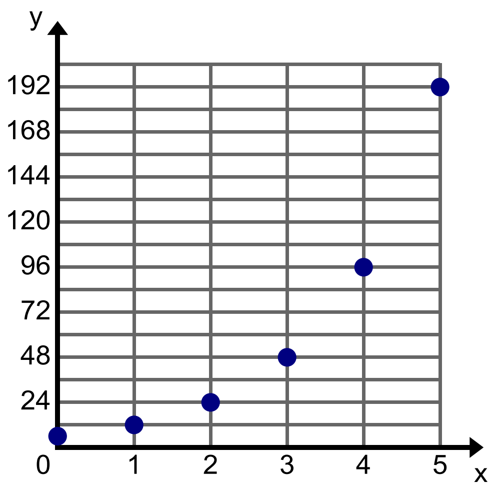
I


y
12
10
8
6
4
2
0
5
10
15
20
25
x

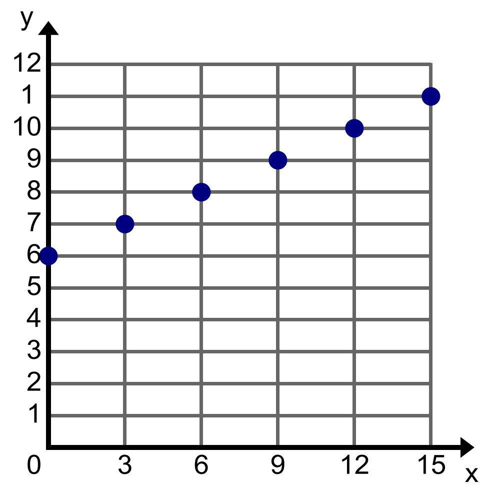
J


y
30
27
24
21
18
15
12
9
6
3
0
1
2
3
4
5
x

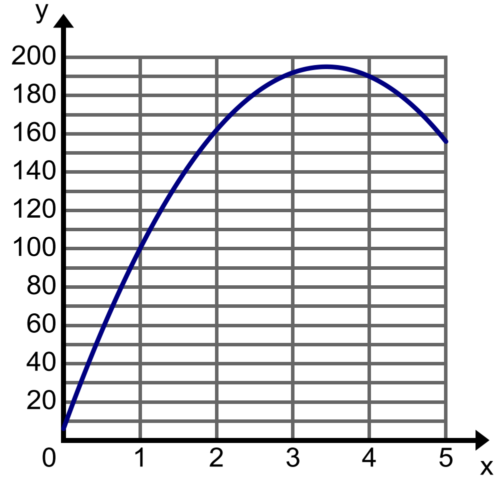
K


y
192
168
144
120
96
72
48
24
0
1
2
3
4
5
x

L

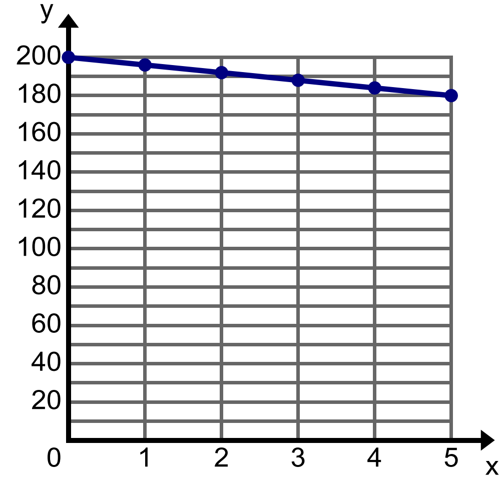

y
12
1
10
9
8
7
6
5
4
3
2
1
0
3
6
9
12
15
x

M



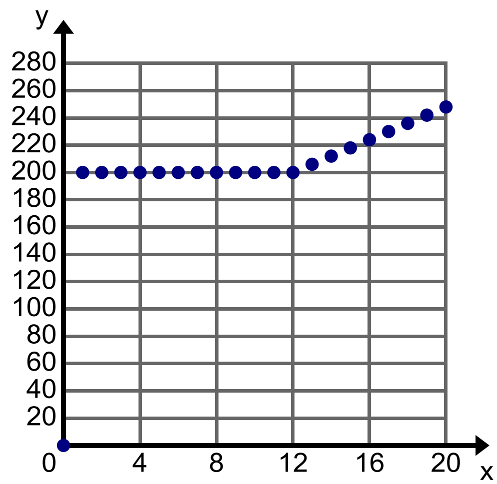
y
200
180
160
140
120
100
80
60
40
20
0
1
2
3
4
5
x

N



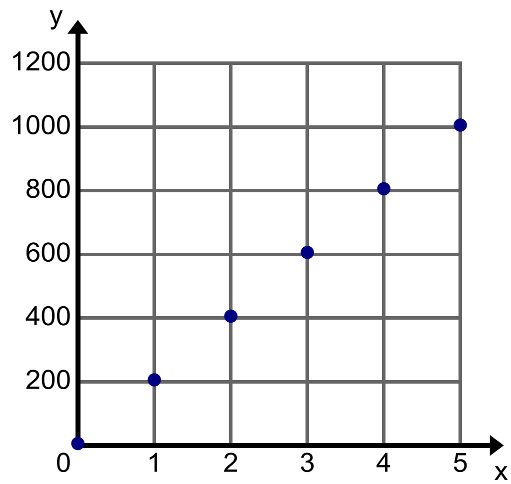
y
200
180
160
140
120
100
80
60
40
20
0
1
2
3
4
5
x

O



y
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
4
8
12
16
20
x

P



y
1200
1000
800
600
400
200
0
1
2
3
4
5
x

Q

$$y = -4x + 200$$

R

$$y = -16x^2 + 110x + 6$$

S

$$y = 3x + 6$$

T

$$y = 200x + 6$$

U

$$y = 6 \cdot 2^x$$

V

$$y = \begin{cases} 200, & \text{if } 0 < x \leq 12 \\ 200 + 6x, & \text{if } x > 12 \end{cases}$$

W

$$y = \frac{1}{5}x + 6$$

X

$$y = \frac{1}{3}x + 6$$

Y
A certain bacteria reproduces by binary fission every hour. This means that one bacterium grows to twice its size, replicates its DNA, and splits in 2. If 6 of these bacterium are placed in a petri dish, how many will there by after 5 hours?

Dependent Variable:
Independent Variable:

Z
The state is building a road 4.5 km long from point A to point B. It takes the crew 3 weeks to complete 600 meters. Six meters of the road have already been completed when this crew starts the job. How much of the road will be completed after 5 weeks?

Dependent Variable:
Independent Variable:

AA
Talen loves to help his mom clean to earn money for his cash box. He currently has $6 in his cash box. He earns $1 for every 3 jobs he does. How much money will Talen have if he does 15 jobs?

Dependent Variable:
Independent Variable:

BB
Josh is draining a swimming pool at a constant rate of 4 gallons per minute. If the swimming pool starts will 200 gallons of water, how many gallons will remain after 5 minutes?

Dependent Variable:
Independent Variable:

CC
Kendall's mom and dad have agreed to sponsor her in a school walk-a-thon to raise money for soccer uniforms. Her mom is donating $6 to her. Her dad is donating $3 for each mile she walks. How much money will she collect if she walks 5 miles?

Dependent Variable:
Independent Variable:

DD
The Planetarium charges $200 for a birthday party for up to 12 guests. Each additional guest is $6. How much will it cost for a birthday party with 20 guests?

Dependent Variable:
Independent Variable:

EE
Suppose a rocket is fired from a platform 6 ft. off the ground into the air vertically with an initial speed of 110 feet/second. Where will the rocket be after 5 seconds? *Note:* The gravitational force of the earth on the rocket is -16 ft. /sec^2.

Dependent Variable:
Independent Variable:

FF
Suzy is helping her mom fill Easter eggs with jelly beans for a community egg hunt. Before they get started, Suzy eats 6 jelly beans. Her mom tells her that after that she can eat 1 jelly bean for every 5 eggs she fills. How many jelly beans total did Suzy eat if she filled 25 eggs?

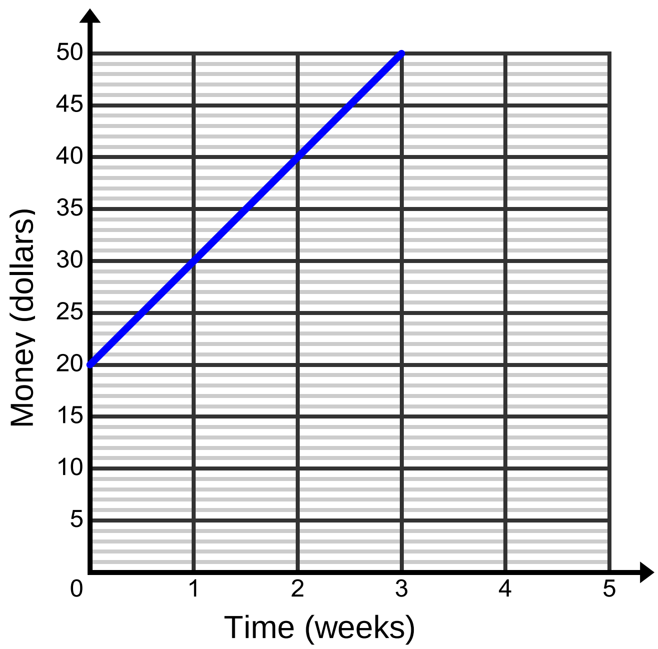
Dependent Variable:
Independent Variable:

4.2e Homework: Comparing Linear Functions

1. Who will have $100 first, George or Mark?

George had $20 and was saving $15 every week.

Mark also started with $20. His savings are shown on the graph below.



2. Put the cyclists in order from slowest to fastest. (Note variables: x = time in seconds, y = meters traveled.)

Cyclist A:

Time (x)	Distance (y)
2	1
4	2
6	3

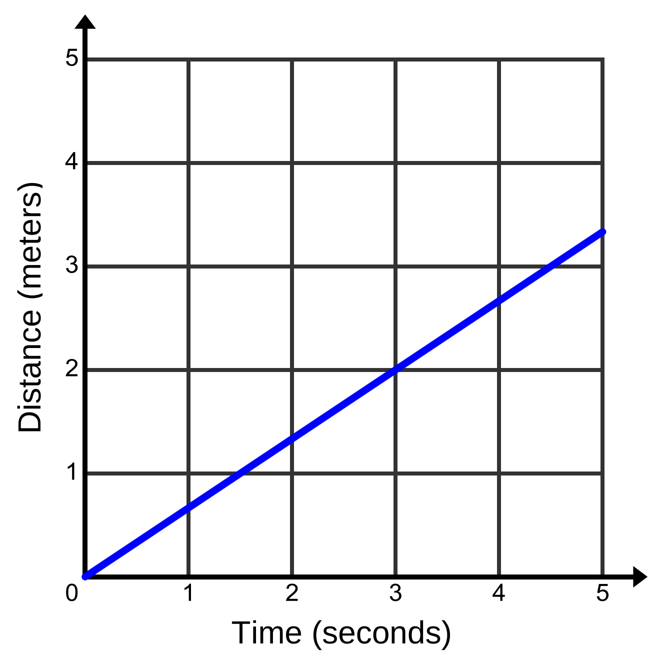
Cyclist B:

Bob has cycled 12 meters in the past 6 seconds.

Cyclist C:

$$y = \frac{1}{3}x$$

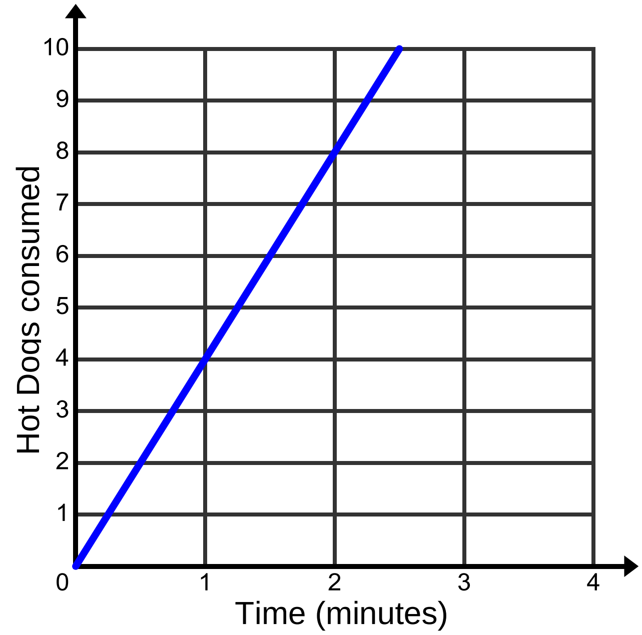
Cyclist D:



3. Assume the rates below will remain consistent. Who will win the hot dog eating contest? Why?

Helga, who has eaten 18 hot dogs in 5 minutes.

Pablo whose eating record is shown below.



4. Based on the information below, which bathtub will be empty first? Why?

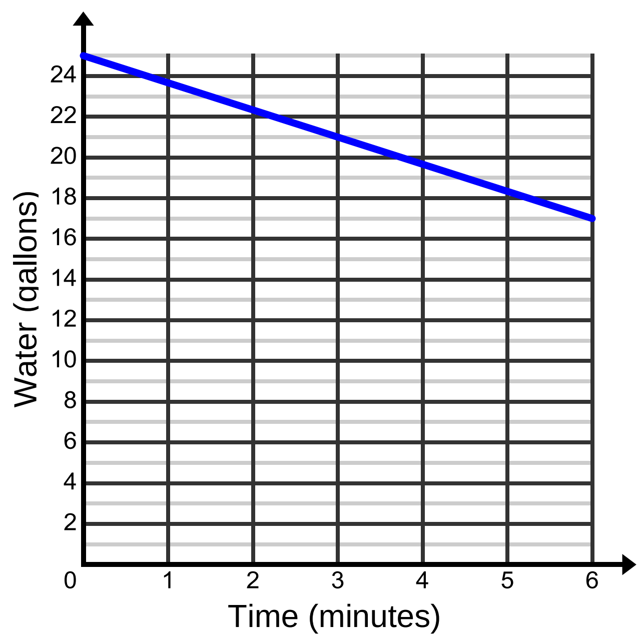
Bathtub A:

Starts with 25 gallons and is draining 1.5 gallons a minute.

Bathtub B:

Minutes	Gallons
0	25
3	20
6	15

Bathtub C:



5. Put the cars in order from fastest to slowest. (Note variables: x = time in hours, y = miles traveled.)

Car A:

$y = 65x$

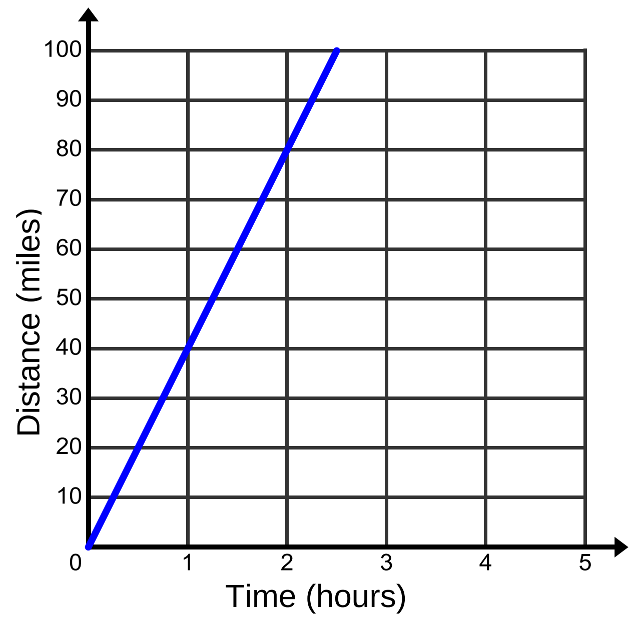
Car B:

The Bakers completed the 64-mile trip to Salt Lake City in 50 minutes.

Car C:

Hours	Miles
2	150
4	300

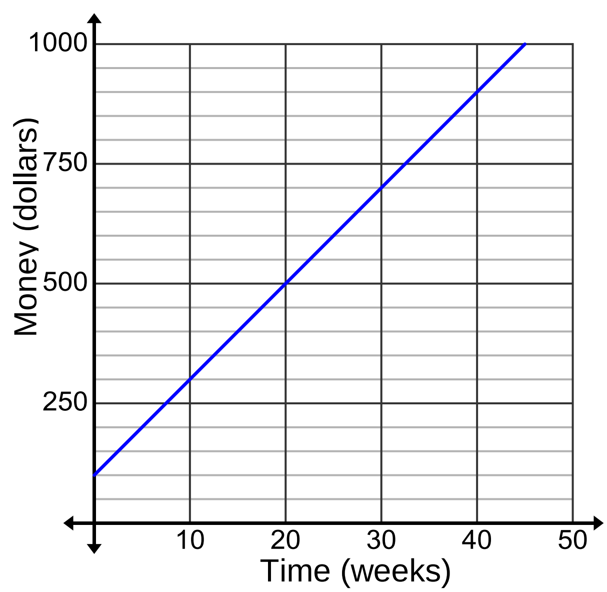
Car D:



6. Who will have $1,000 first, Becky or Olga?

Becky had $100 and was saving $10 every week.

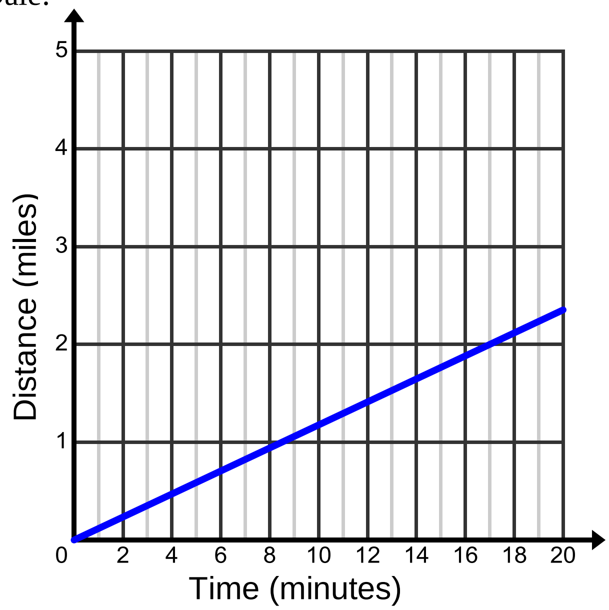
Olga's information is shown on the graph below.



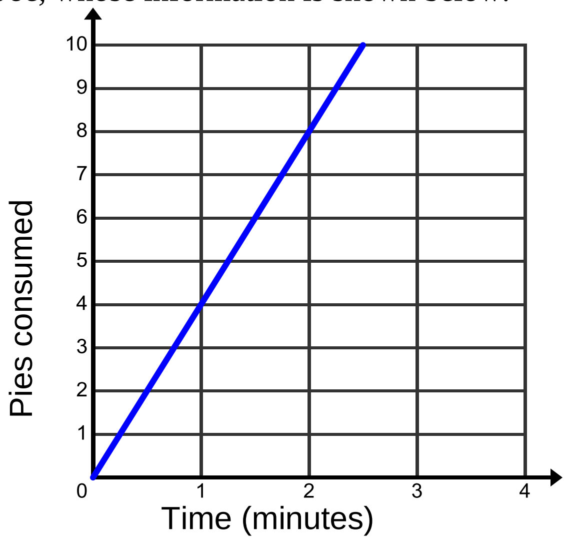
7. Put the runners in order from slowest to fastest. (Note variables: x = time in minutes, y = miles traveled.)

<table>
<tr><td>Ellen ran 1 mile in the last 10 minutes</td><td>Samantha:
$y = \frac{2}{13}x$</td></tr>
<tr><td>Jason:

Time (minutes)	Distance (miles)
3	0.5
6	1
9	1.5

</td><td>Dale:

</td></tr>
</table>

8. Assume the rates below will remain consistent. Who will win the pie eating contest? Why?

<table>
<tr><td>Joe, whose information is shown below.

</td><td>Donna, who has eaten 11 pies in 2.5 minutes.</td></tr>
</table>

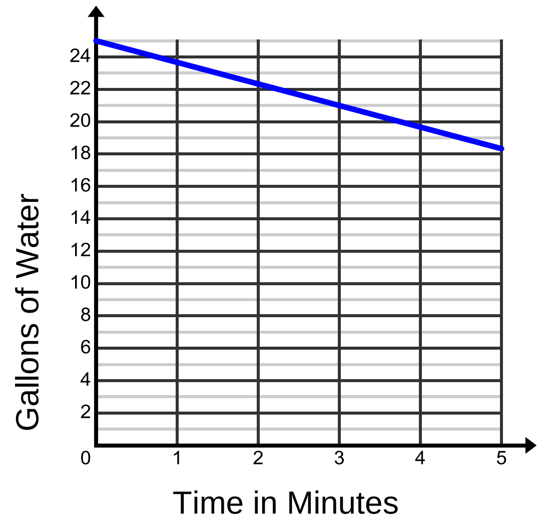
9. Based on the information below, which hot water heater will use up the available hot water first?

Water Heater A:
Starts with 50 gallons and drains 1.5 gallons a minute.

Water Heater B:

Time in Minutes	Gallons
0	30
2	27.5
4	25

Water Heater C:



10. Put the cars in order from fastest to slowest. (Note variables: x = time in hours, y = miles traveled.)

Car A:
$y = 35x$

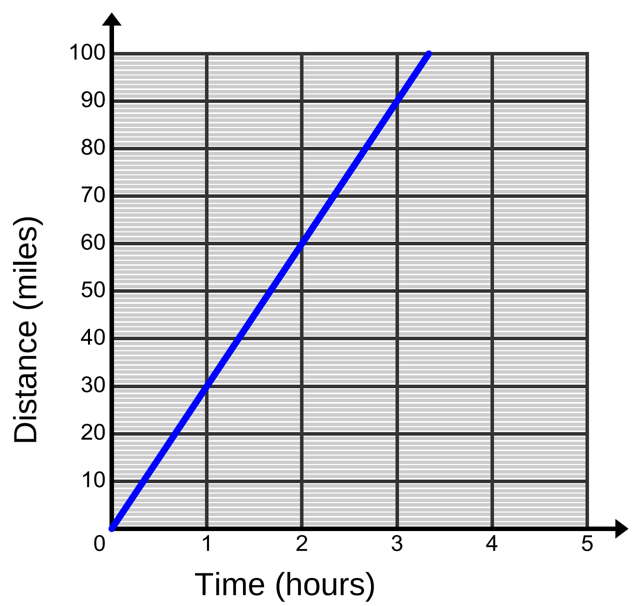
Car B:

The Andersons completed the 24 mile trip to Salt Lake City in 30 minutes.

Car C:

Time (hours)	Distance (miles)
0.5	18
2.5	90
4	144

Car D:



4.2f Self-Assessment: Section 4.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Deep Understanding, Skill Mastery
1. Distinguish between linear and nonlinear functions given a context, table, graph, or equation.			
2. Understand how a linear function grows (changes).			
3. Match the representations (table, graph, equation, and context) of linear and nonlinear situations.			
4. Compare properties of linear functions (rates of change and intercepts) and use this information to solve problems.			

Section 4.3: Model and Analyze a Functional Relationship

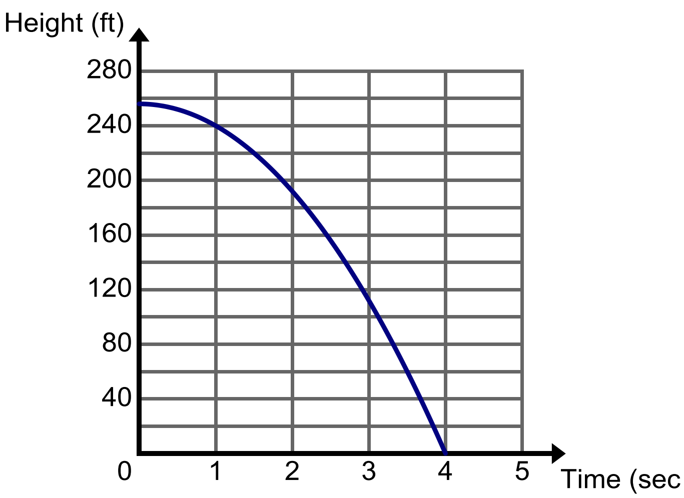
Section Overview:
In this section, students will construct functions to model the relationship between two quantities that are linearly related. They will also describe the functional relationship between two quantities by describing and analyzing features of the graph. They will then sketch graphs that display key features of a function given a verbal description of the relationship between two quantities.

Concepts and Skills to Master:
By the end of this section, students should be able to:
1. Determine whether the relationship between two quantities can be modeled by a linear function.
2. Construct a function to model a linear relationship between two quantities.
3. Identify and interpret key features of a graph that models a relationship between two quantities.
4. Sketch a graph that displays key features of a function that has been described verbally.

4.3a Classwork: Constructing Linear Functions

Directions: Identify the dependent and independent variable in the following situations. Determine whether the situations are linear or nonlinear. **For the situations that are linear,** construct a function that models the relationship between the two quantities. Be sure to define your variables.

1. An object is dropped from a bridge into the water below. The graph below shows the height of the object (in feet) with respect to time (in seconds). Consider the relationship between the height of the object and time.



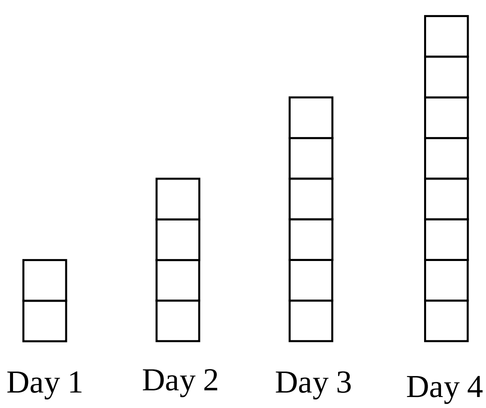
Independent Variable:

Dependent Variable:

Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two quantities. Be sure to define your variables.

2. Owen is earning pennies each day that he makes his bed in the morning. On the first day, Owen's mom gives him 2 pennies. On the second day, Owen's mom gives him 4 pennies, on the third day 6 pennies, on the fourth day 8 pennies. Owen makes his bed every day and this pattern continues. The model below shows how many pennies Owen earns each day (each box represents 1 penny). Consider the relationship between **the number of pennies received on a given day** and the **day number**.



Independent Variable:

Dependent Variable:

Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

3. Refer back to #2 and Owen earning pennies. Consider the relationship between the **total number of pennies** Owen has earned and the **day number**.

Independent Variable:

Dependent Variable:

Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

4. Carbon-14 has a half-life of 5,730 years. The table below shows the amount of carbon-14 that will remain after a given number of years. Consider the relationship between number of years and amount of carbon-14 remaining.

# of Years	**Milligrams of Carbon-14**
0	8
5,730	4
11,460	2
17,190	1
22,920	$\frac{1}{2}$

Independent Variable:

Dependent Variable:

Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

5. The table below shows the amount of time a recipe recommends you should roast a turkey at 325˚ F dependent on the weight of the turkey in pounds. Consider the relationship between cooking time and weight of the turkey.

Weight of Turkey (lbs.)	12	13	14	15
Cooking Time (hours)	4	$4\frac{1}{3}$	$4\frac{2}{3}$	5

Independent Variable:

Dependent Variable:

Is the data linear? Why or why not?

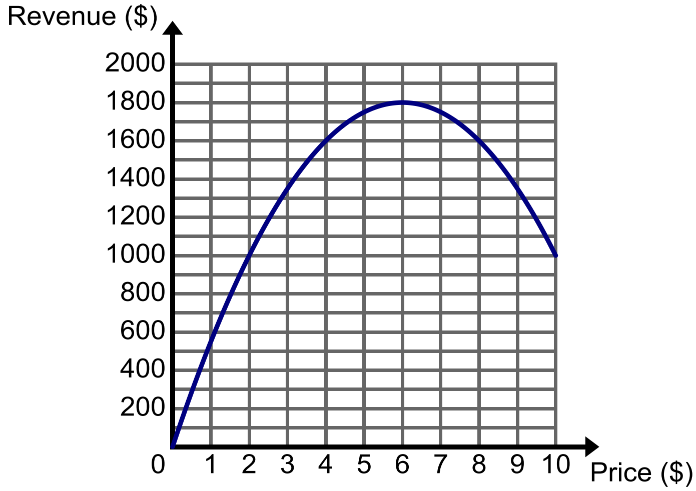
If yes, construct a function to model the relationship between the two variables. Be sure to define your variables.

6. Steve is a lifeguard at a local community pool. Each day at noon, he records the temperature and the number of people in the pool. Do you think the relationship between temperature and number of people in the pool is linear? Why or why not?

4.3a Homework: Constructing Linear Functions

Directions: Determine whether the situations represented below are linear or nonlinear. **For the situations that are linear,** construct a function that models the relationship between the two quantities. Be sure to define your variables.

1. The graph below shows the amount of revenue a company will make selling t-shirts dependent on the price of each t-shirt. Consider the relationship between price of each shirt and revenue made.



Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables. If no, describe why the graph looks the way it does.

2. When Camilo opened his email this morning he had 140 unread emails. The table below shows the number of remaining unread emails Camilo has in his inbox. Assume that Camilo does not receive any new emails while he is reading his email. Consider the relationship between time and the number of unread emails.

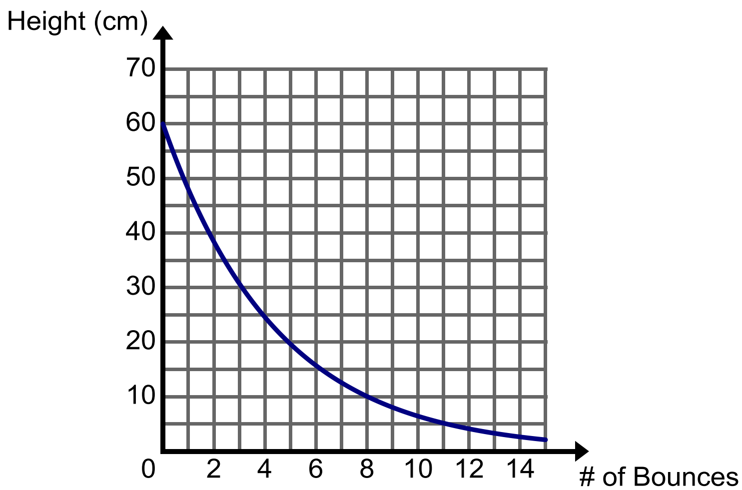
Time (hours)	# of Unread Emails
0	180
0.5	160
1	140
2	100
2.5	80
4	20
4.5	0

Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two quantities. Be sure to define your variables.

3. Suppose a certain bank pays 4% interest at the end of each year on the money in an account. When Devon was born, his parents put $100 in the account and will leave it there until he goes to college. Is the relationship between time (in years) and the amount of money in the account (in dollars) linear or not? Why or why not? If it is linear, write a function that models the relationship between the two quantities.

4. Suppose you drop a basketball from a height of 60 inches. The graph below shows the height of the object after *b* bounces.



Is the data linear? Why or why not?

If yes, construct a function to model the relationship between the two variables. Be sure to define your variables. If no, describe why the graph looks the way it does.

5. Justine and her family are floating down a river. After 1 hour, they have floated 1.25 miles, after 4 hours they have floated 5 miles, and after 6 hours they have floated 7.5 miles. Is the relationship between time (in hours) and distance (in miles) linear? Why or why not? If it is linear, write a function that models the relationship between the two quantities.

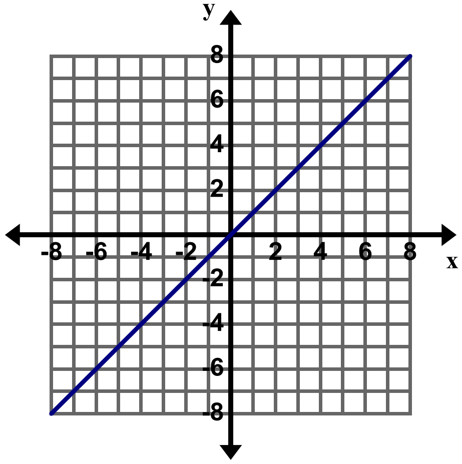
6. You and your friends go to a BMX dirt-biking race. For one of the events, the competitors are going off a jump. The winner of the event is the competitor that gets the most air (or jumps the highest). Do you think the relationship between the weight of the bike and the height of the jump can be modeled by a linear relationship? Why or why not?

7. Homes in a certain neighborhood sell for $117 per square foot. Can the relationship between the number of square feet in the home and the sale price of the home be modeled by a linear function? Why or why not? If it can be modeled by a linear function, write a function that models the relationship between the two quantities.

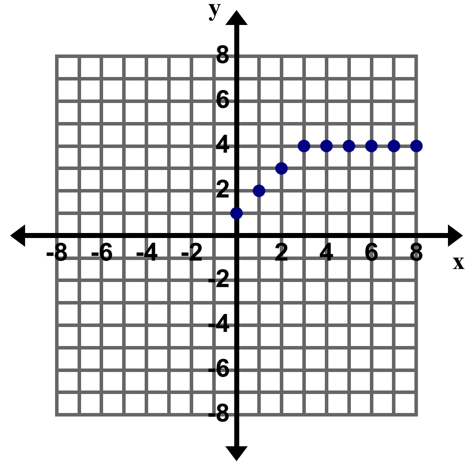
4.3b Class Activity: Features of Graphs

1. Cut out each graph. Sort the graphs into groups and be able to explain why you grouped the graphs the way you did. In the table that follows, name your groups, describe your groups, and list the graphs that are in your group.

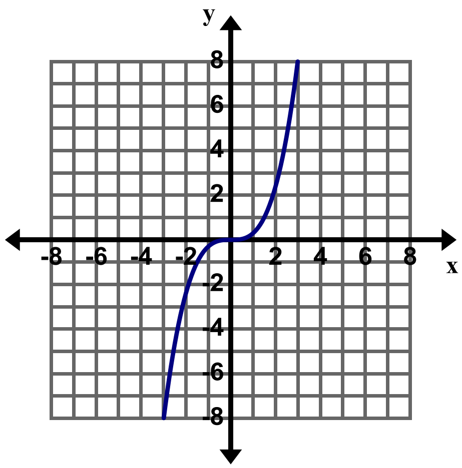
A



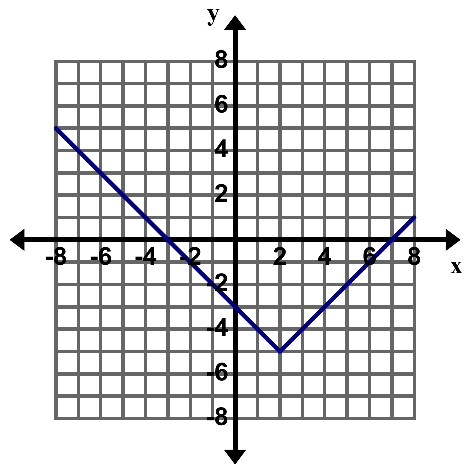
B



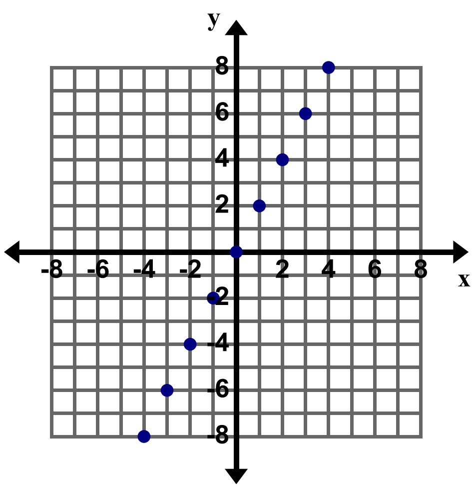
C



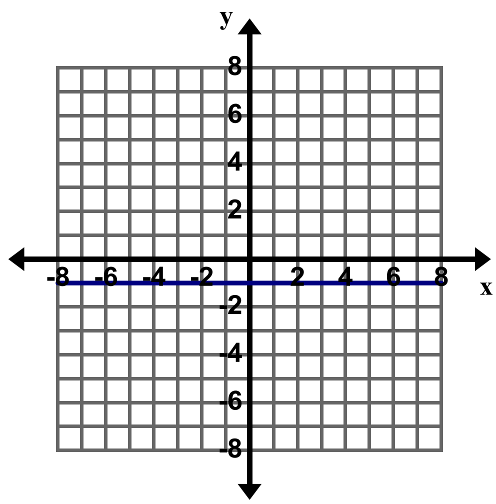
D



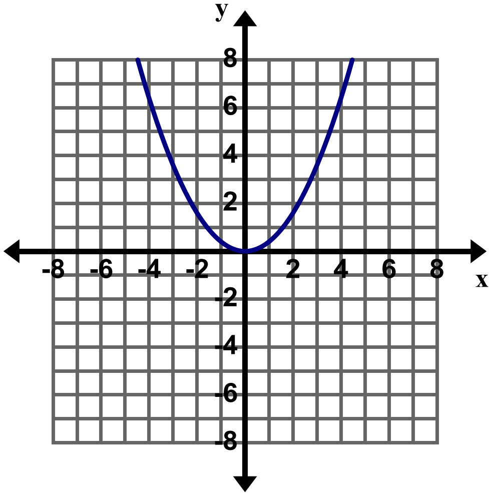
E



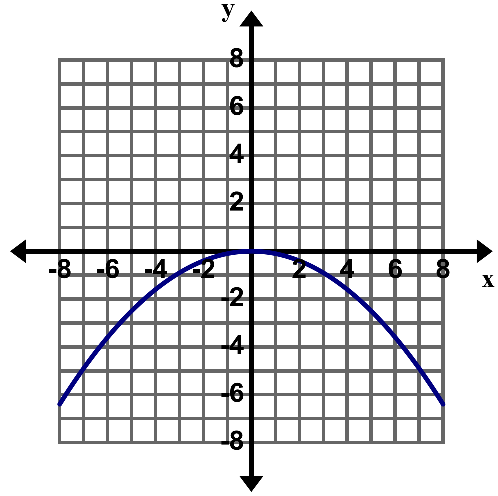
F



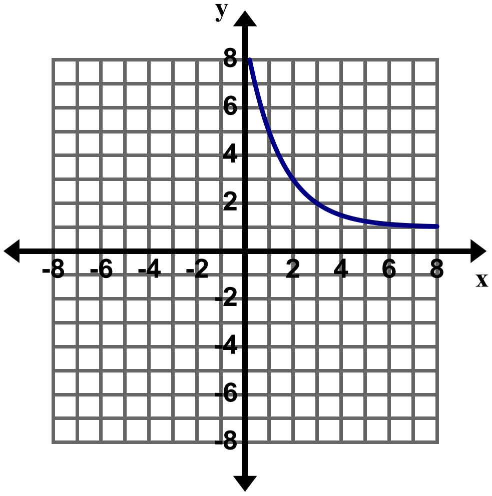
G


y
8
6
4
2
-8 -6 -4 -2 2 4 6 8
x
-2
-4
-6
-8

H


y
8
6
4
2
-8 -6 -4 -2 2 4 6 8
x
-2
-4
-6
-8

I


y
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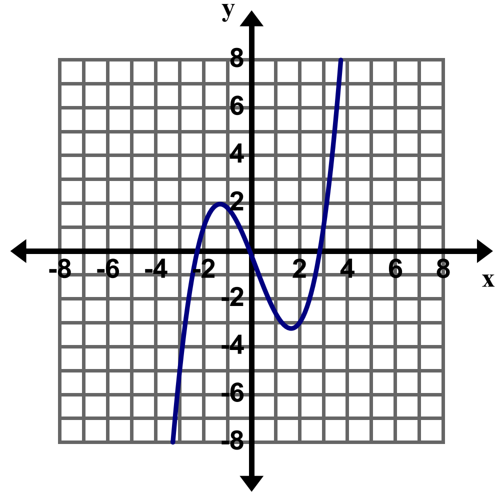
6
4
2
-8 -6 -4 -2 2 4 6 8
x
-2
-4
-6
-8

J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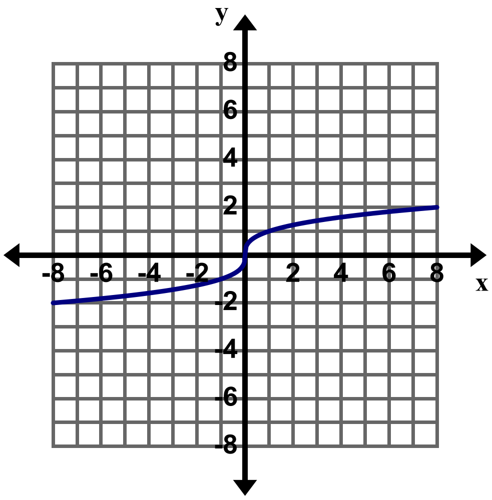
6
4
2
-8 -6 -4 -2 2 4 6 8
x
-2
-4
-6
-8

K


y
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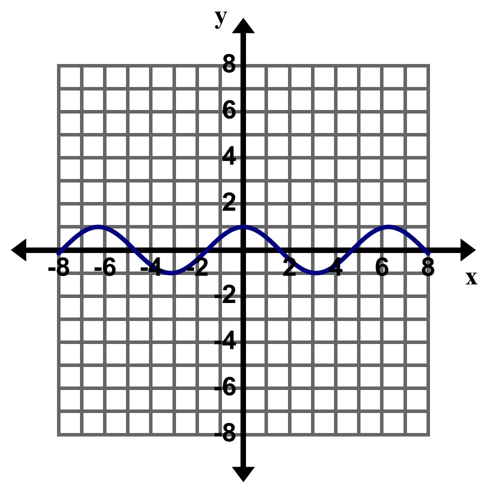
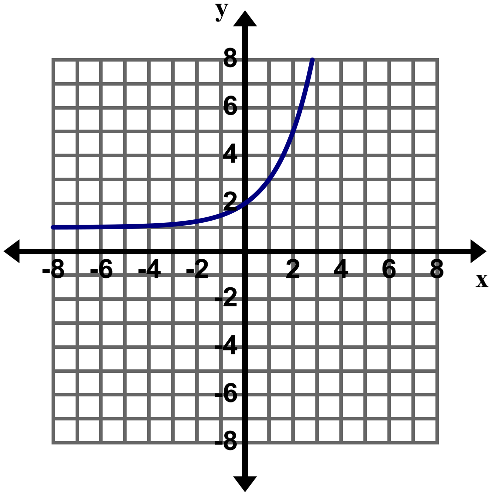
6
4
2
-8 -6 -4 -2 2 4 6 8
x
-2
-4
-6
-8

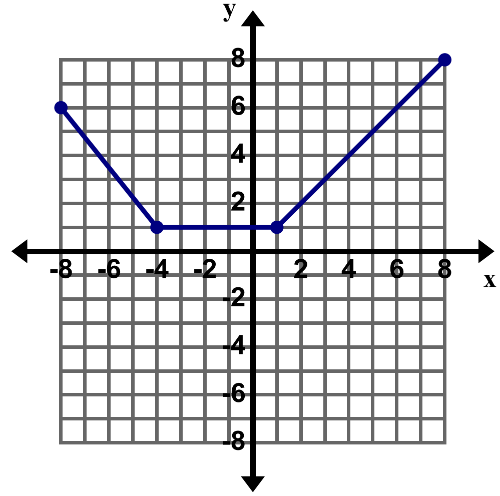
L


y
8
6
4
2
-8 -6 -4 -2 2 4 6 8
x
-2
-4
-6
-8

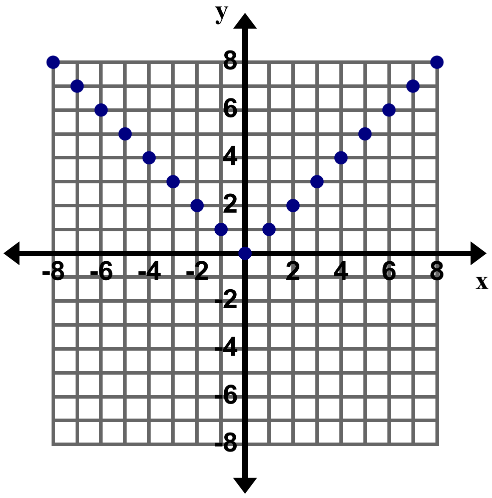
M


y
8
6
4
2
-8 -6 -4 -2 2 4 6 8
x
-2
-4
-6
-8

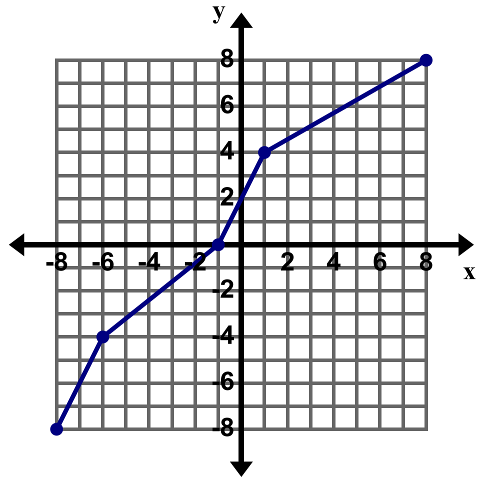
N


y
8
6
4
2
-8 -6 -4 -2 2 4 6 8
x
-2
-4
-6
-8

O


y
8
6
4
2
-8 -6 -4 -2 2 4 6 8
x
-2
-4
-6
-8

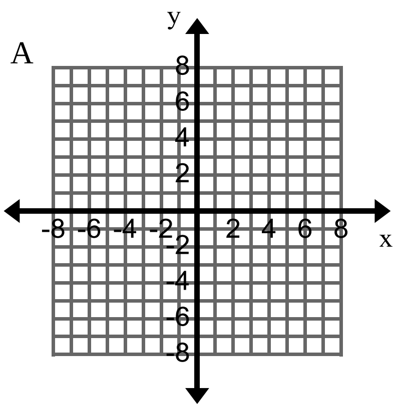
P

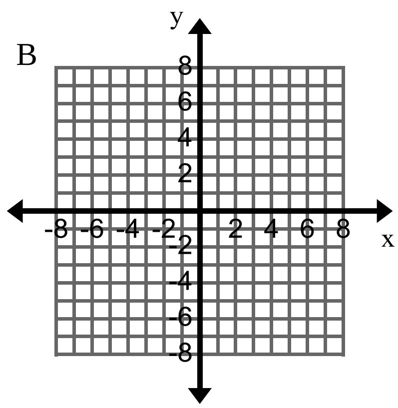

y
8
6
4
2
-8 -6 -4 -2 2 4 6 8
x
-2
-4
-6
-8

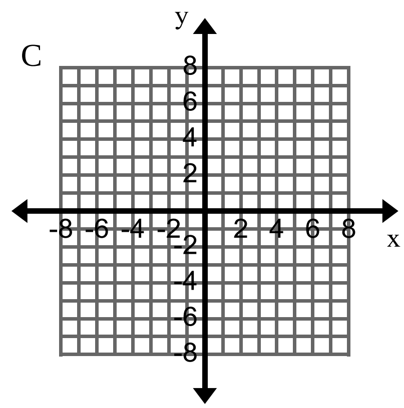
Your Groups

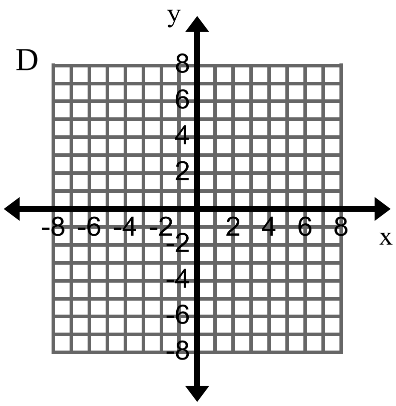
Name of the group	Description of the group	Graphs in the group
A.		
B.		
C.		
D.		
E.		
F.		

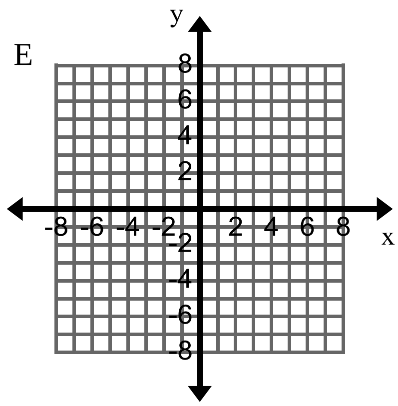
2. For each group that you created, draw another graph that would fit in that group.

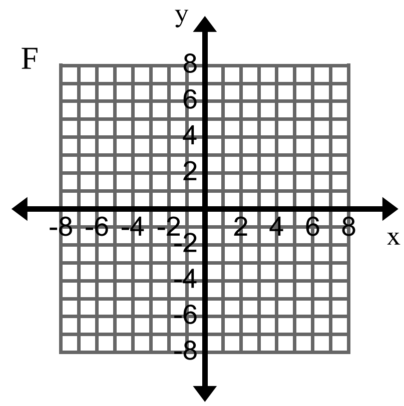
A


B


C


D


E


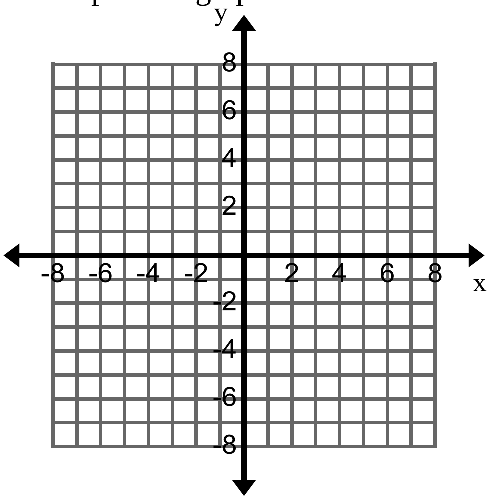
F


3. Lucy grouped hers as follows:

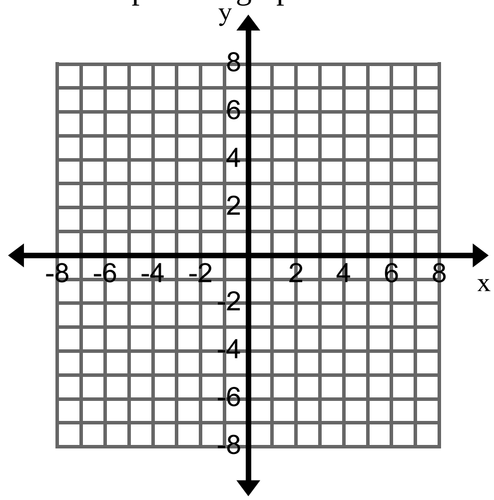
Increasing on the entire graph: **A, C, E, K, M, P**
Decreasing on the entire graph: **I**
Constant on the entire graph: **F**
Increasing on some parts of the graph, decreasing on some parts of the graph: **D, G, H, J, L, O**
Increasing on some parts of the graph, decreasing on some parts of the graph, constant on some parts of the graph: **N**
Increasing on some parts of the graph, constant on some parts of the graph: **B**

4. Define **increasing**, **decreasing**, and **constant** in your own words.

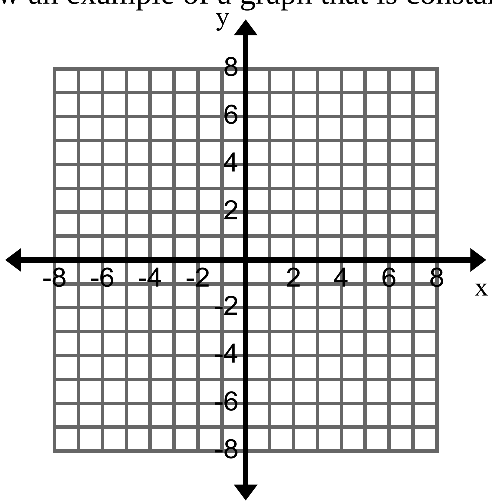
5. Draw an example of a graph that increases.



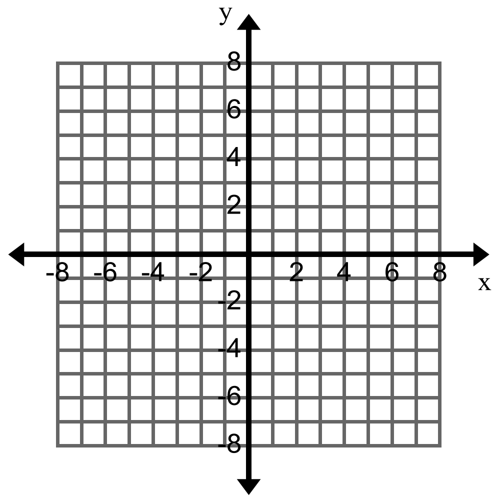
6. Draw an example of a graph that decreases.



7. Draw an example of a graph that is constant.



8. Draw an example of a graph that increases and decreases.

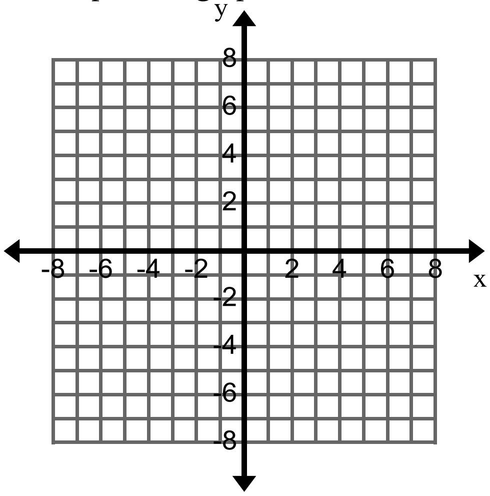


9. Ellis grouped hers as follows:

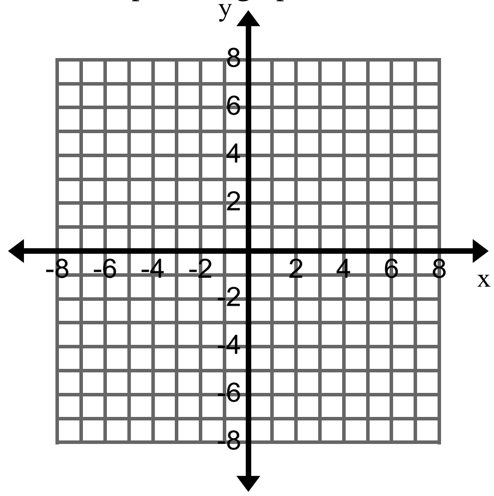
Discrete: **B, E, O**
Continuous: **A, C, D, F, G, H, I, J, K, L, M, N, P**

10. Define **discrete** and **continuous** in your own words. Can you think of a real world situation that has a discrete graph? Why doesn't it make sense to connect the points in this situation?

11. Draw an example of a graph that is discrete.



12. Draw an example of a graph that is continuous.



13. Grace grouped hers as follows:

Linear: **A, E, F**
Nonlinear: **C, G, H, I, J, K, L, M**
Made up of pieces of different linear functions: **B, D, N, O, P**

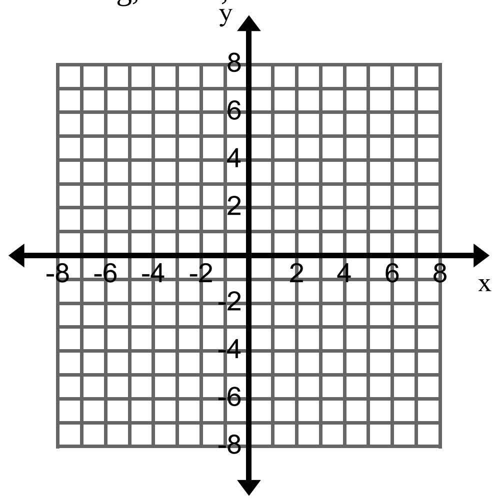
14. Define **linear** in your own words.

15. Define **nonlinear** in your own words.

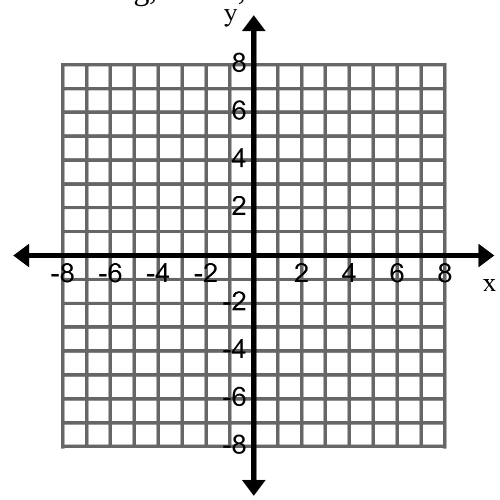
4.3b Homework: Features of Graphs

Directions: Draw a graph with the following features.

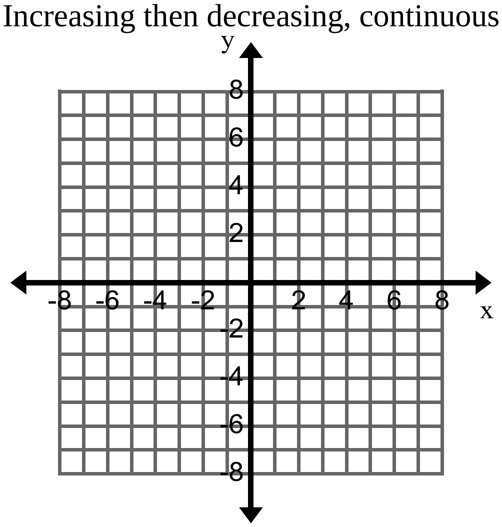
1. increasing, linear, continuous



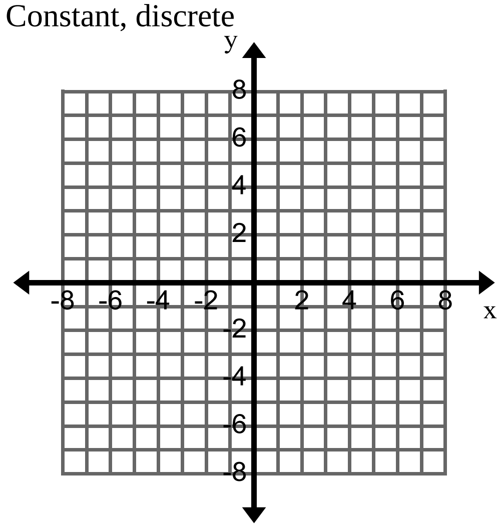
2. Decreasing, linear, and discrete



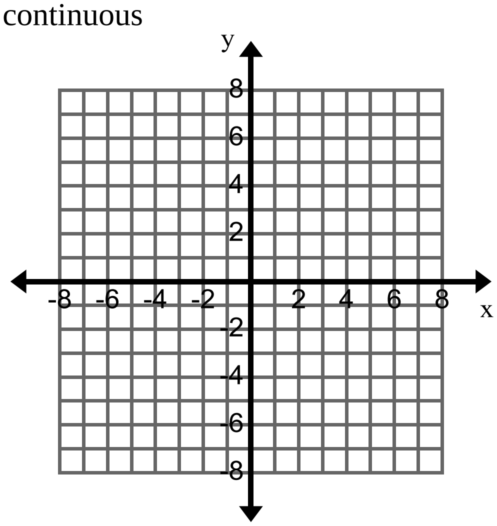
3. Increasing then decreasing, continuous



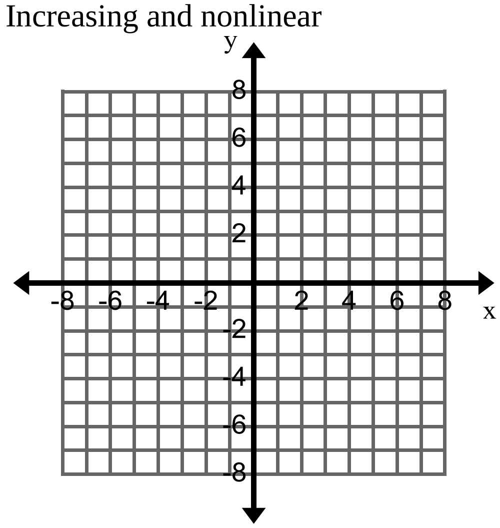
4. Constant, discrete



5. Decreasing, constant, then increasing, continuous

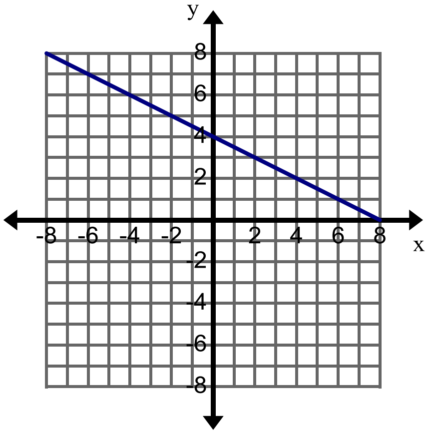


6. Increasing and nonlinear



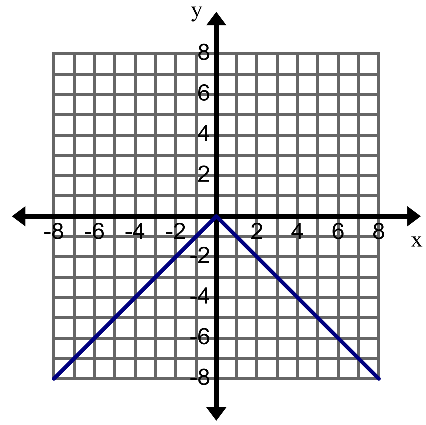
Directions: Describe the features of each of the following graphs (increasing/decreasing/constant; discrete/continuous; linear/nonlinear). Label on the graph where it is increasing, decreasing, or constant. Identify the intercepts of the graph.

7.



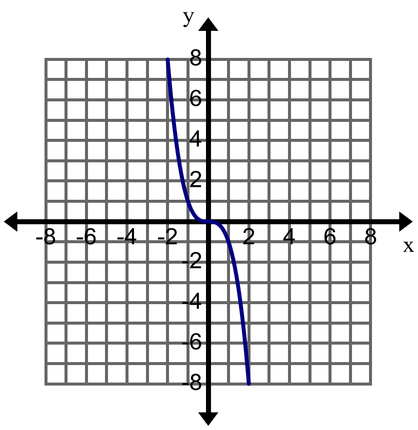
Features:

8.



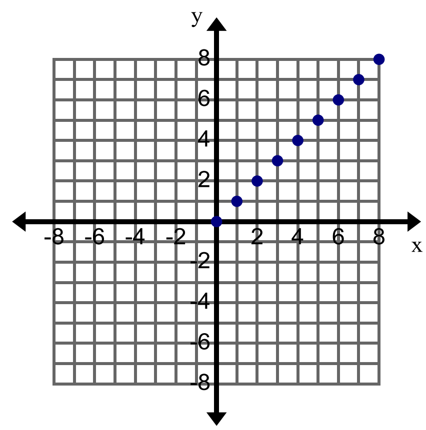
Features:

9.



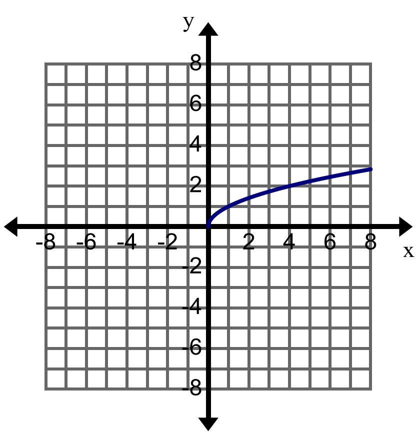
Features:

10.



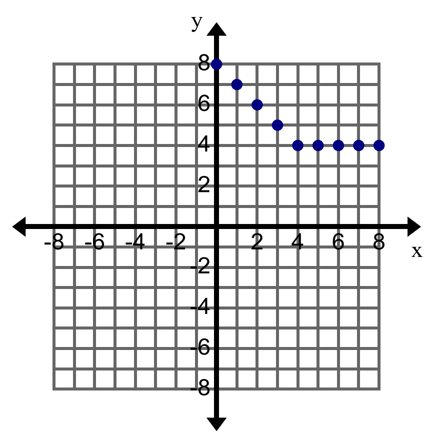
Features:

11.



Features:

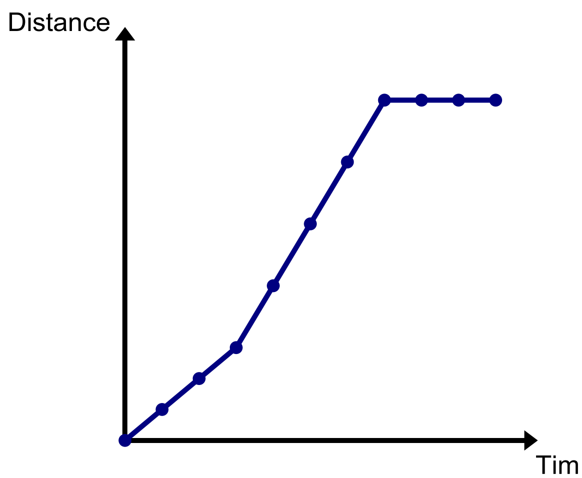
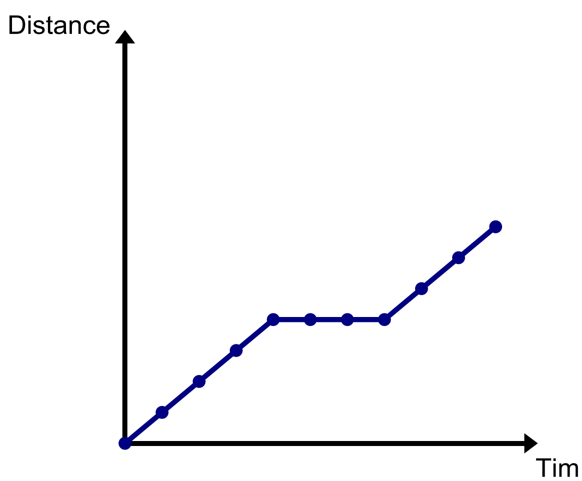
12.

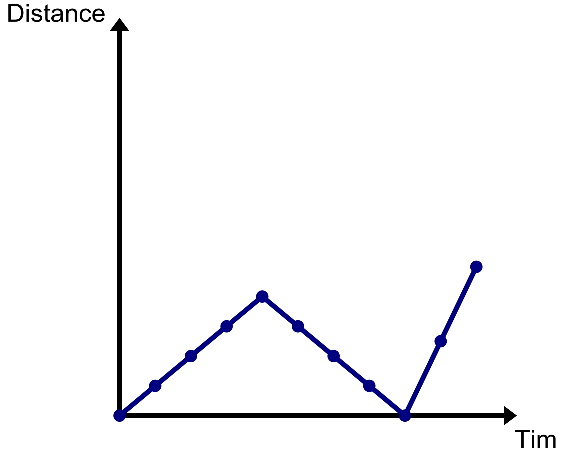
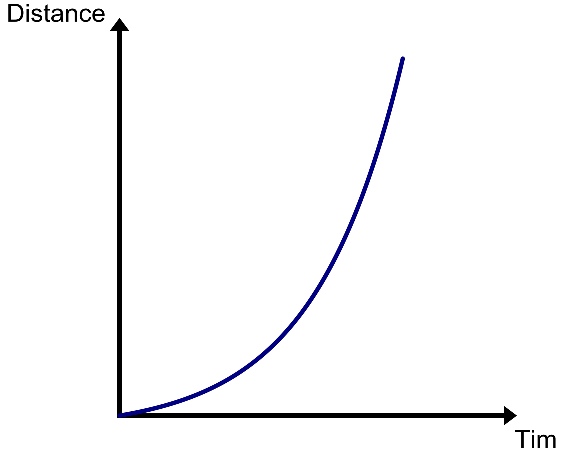
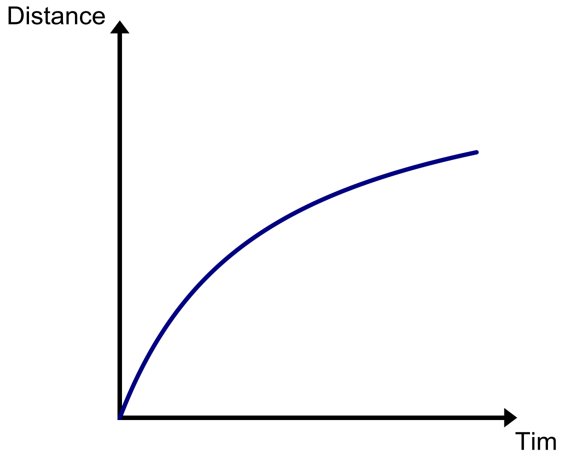


Features:

4.3c Classwork: School's Out

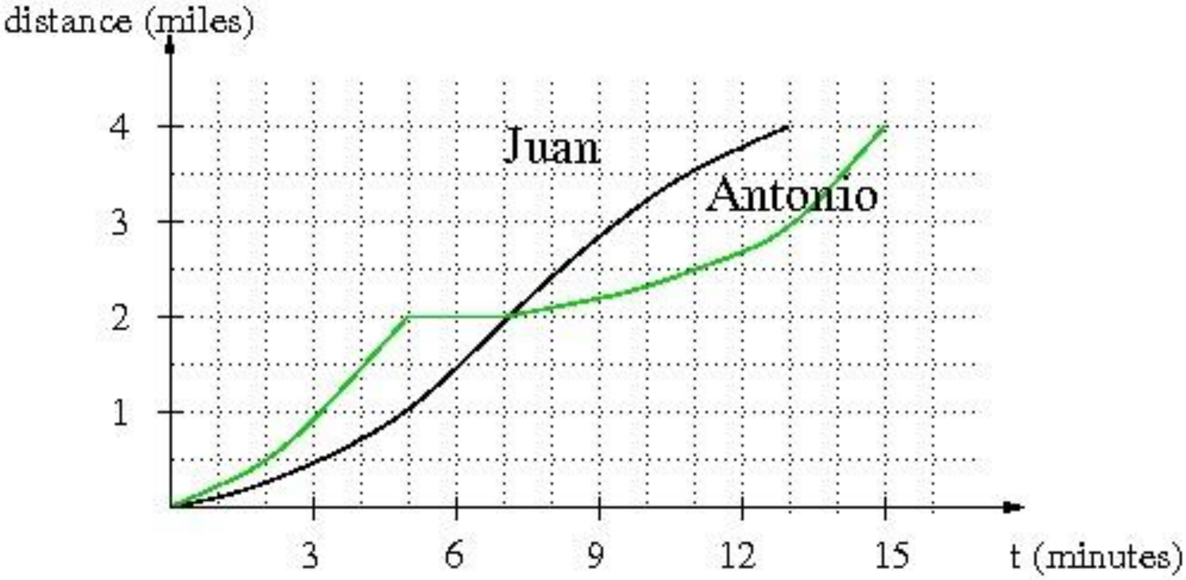
Directions: The following graphs tell the story of five different students leaving school and walking home. Label the key features of the graph. Write a story for each graph describing the movement of each of the students.

Abby's Journey Home: 	
Beth's Journey Home: 	

Chad's Journey Home:  	
Drew's Journey Home:  	
Eduardo's Journey Home:  	

4.3c Homework: Bike Race

Antonio and Juan are in a 4-mile bike race. The graph below shows the distance of each racer (in miles) as a function of time (in minutes).

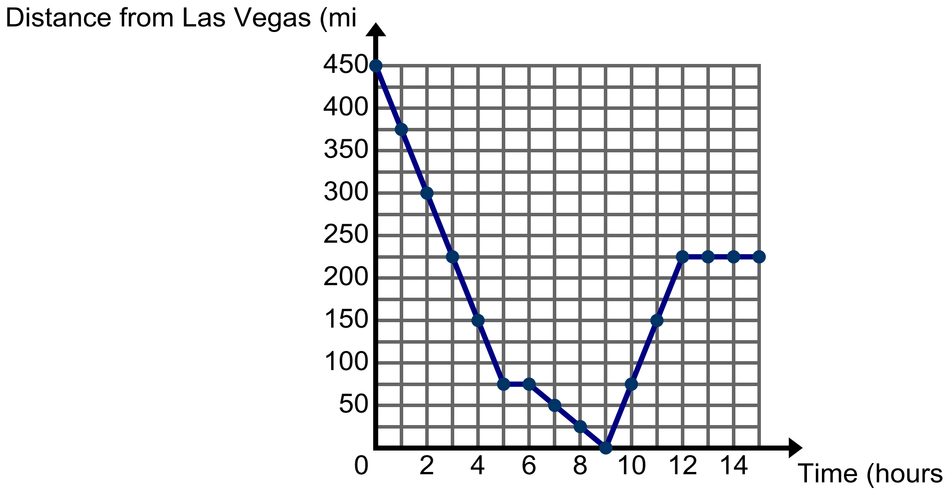


1. Who wins the race? How do you know?

2. Imagine you were watching the race and had to announce it over the radio, write a story describing the race.

4.3d Classwork: From Graphs to Stories

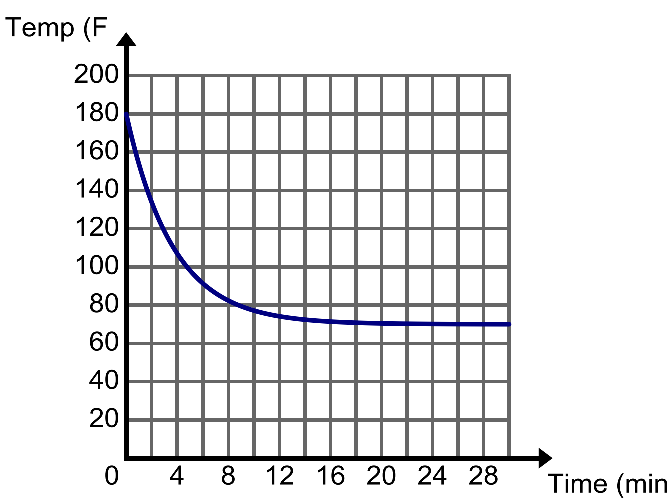
1. Ben and his family took a road trip to visit their cousins. The graph below shows their journey. Label the key features of the graph.



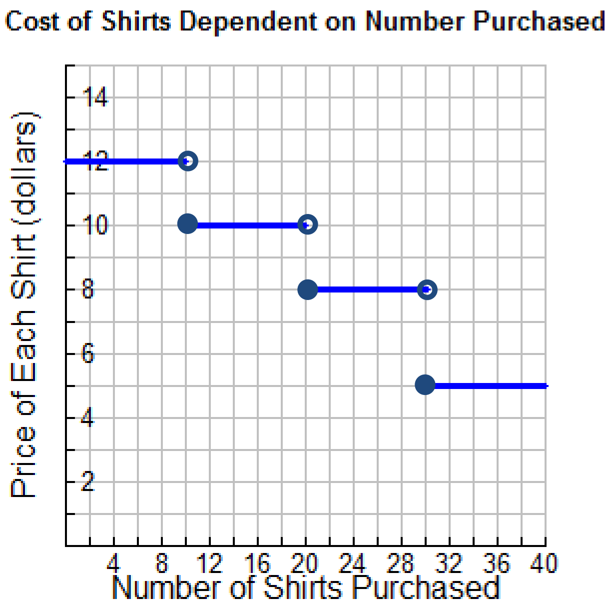
Write a story about the graph.

2. Cynthia is doing research on how hot coffee is when it is served. At one coffee shop, the coffee was served at 180˚ F. Cynthia bought a coffee and then left it on the counter to cool. The temperature in the room was 70˚ F. The graph below shows the temperature of the coffee (in ˚F) as a function of time (in minutes) since it was served. Label the key features of the graph.

Write a story about the graph.

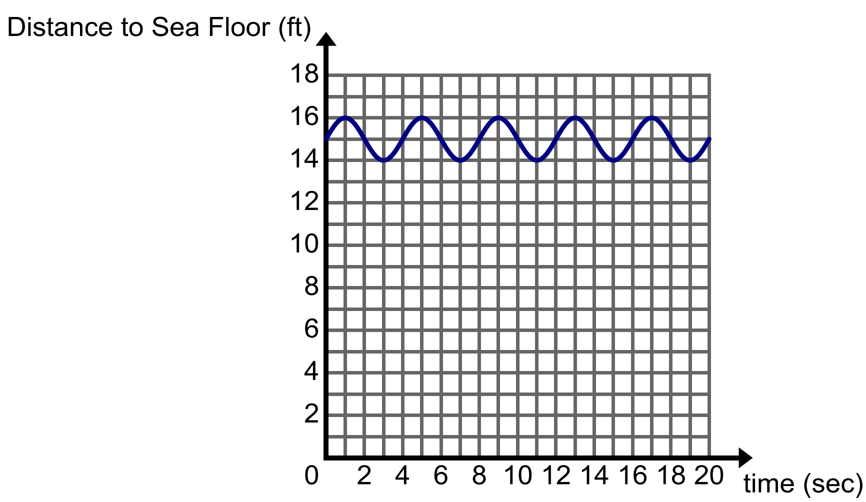


3. Jorge is the team captain of his soccer team. He would like to order shirts for the team and is looking into how much it will cost. He called Custom T’s to ask about pricing and the manager sent him the following graph.



Help Jorge to interpret this graph by telling the story of the graph in the space below.

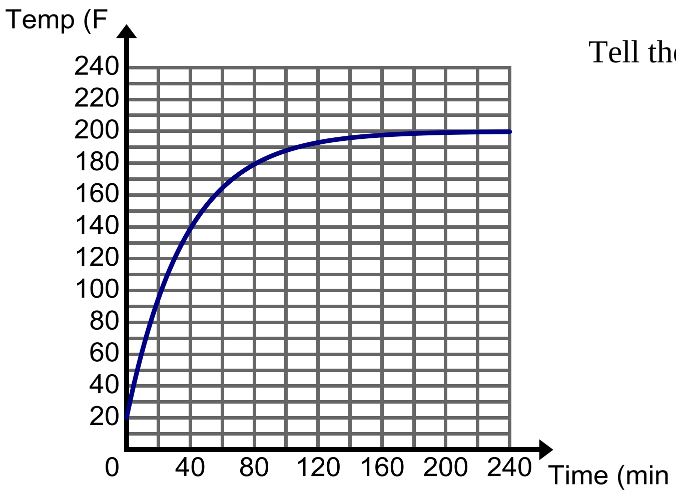
4. A boat is anchored near a dock. The graph below shows the distance from the bottom of the boat to the sea floor over a period of time.



Write a story about the graph.

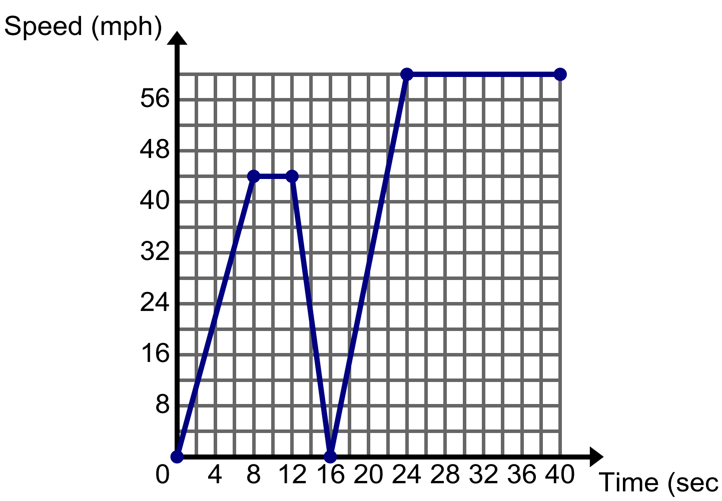
4.3d Homework: From Graphs to Stories

1. Tessa is cooking potatoes for dinner. She puts some potatoes in an oven pre-heated to 200˚ F. The graph below shows the temperature of the potatoes over time. Label the key features of the graph. The *y*-intercept of the graph is (0, 20).



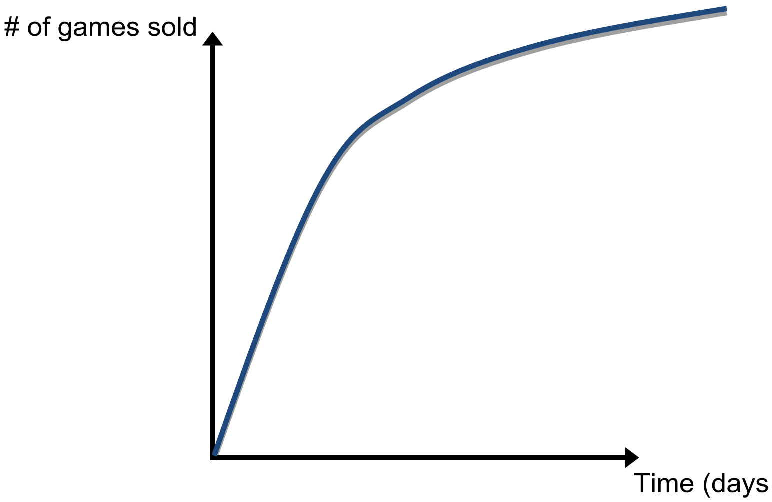
Tell the story of this graph.

2. Steve is driving to work. The graph below shows Steve's speed over time. Label the key features of the graph.



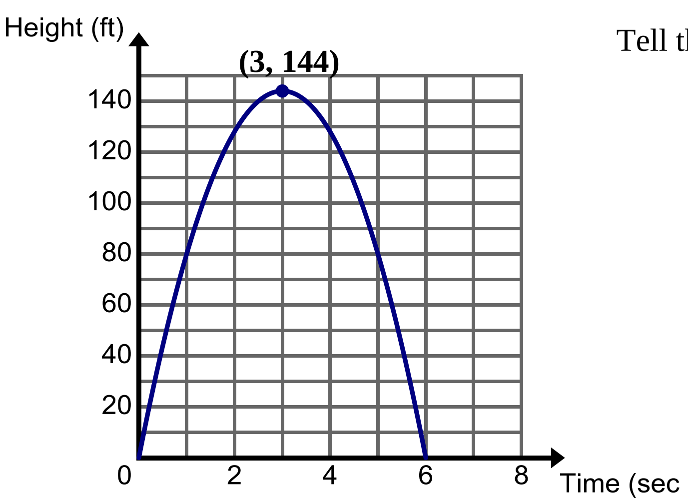
Tell the story of this graph.

3. Microsoft is releasing the most anticipated new Xbox game of the summer. The graph below shows the total number of games sold as a function of the number of days since the game was released.



Tell the story of this graph.

4. A toy rocket is launched straight up in the air from the ground. It leaves the launcher with an initial velocity of 96 ft. /sec. The graph below shows the height of the rocket in feet with respect to time in seconds. Label the key features of the graph.



Tell the story of this graph.

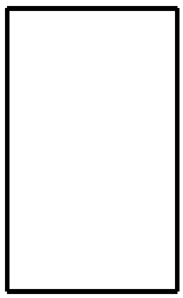
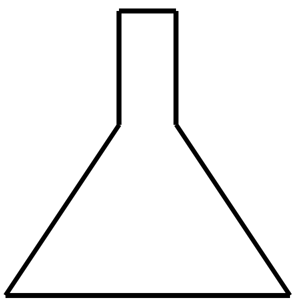
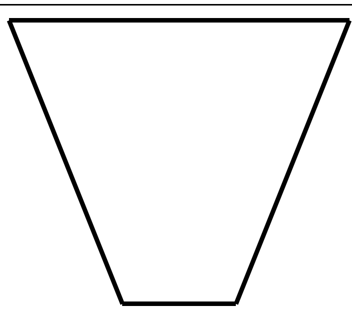
4.3e Classwork: From Stories to Graphs

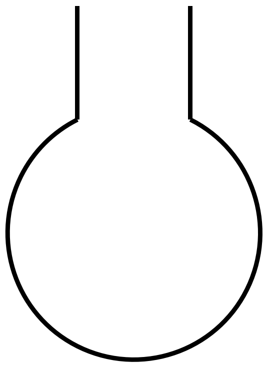
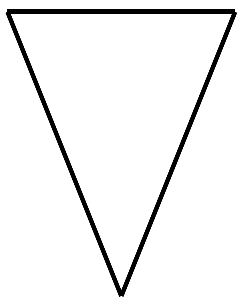
Directions: Sketch a graph to match each of the following stories. Label key features of your graph.

Story	Graph
1. Zach walks home from school. When the bell rings, Zach runs to his locker to grab his books and starts walking home. When he is about halfway home, he realizes that he forgot his math book so he turns around and runs back to school. After retrieving his math book, he realizes that he is going to be late so he sprints home. Sketch a graph of Zach's distance from school as a function of time since the bell rang.	
2. Solitude is offering a ski clinic for teens. The cost of the class is $30 per student. A minimum of 5 students must sign up in order for Solitude to hold the class. The maximum number of students that can participate in the class is 12. Sketch a graph that shows the revenue Solitude will bring in dependent on the number of students that take the class.	
3. A biker is riding up a hill at a constant speed. Then he hits a downhill and coasts down the hill, picking up speed as he descends. At the bottom of the hill, he gets a flat tire. Sketch a graph that shows the distance traveled by the biker as a function of time.	

4. A concert for a popular rock group is sold out. The arena holds 8,000 people. The rock group is scheduled to take the stage at 8 pm. A band that is not very well known is opening for the rock band at 6:30 pm. The rock band is scheduled to play for 2 hours and the staff working the concert have been told that the arena must be cleared of people by 11:30 pm. Sketch a graph of the number of people in the arena from 5 pm to midnight.

5. Your science teacher has the beakers shown below. He is going to fill them with water from a faucet that runs at a constant rate. Your job is to sketch a graph of the height of the water in each of the beakers over time.

Beaker	Graph of the height of the water over time
 **Beaker 1**	
 **Beaker 2**	
 **Beaker 3**	

 **Beaker 4**	
 **Beaker 5**	

6. Now consider the volume of the water in each of the beakers over time. Sketch a graph of the volume of the water in each of the beakers over time.

4.3e Homework: From Stories to Graphs

Directions: Sketch a graph for each of the stories below.

1. Sketch a graph of the number of students in the cafeteria as a function of time throughout the school day at your school.

2. Two thousand, five hundred students attend a local high school. School starts at 8 am and ends at 2:30 pm. Many students stay after school for clubs, sports, etc. The school has a one-hour lunch at noon and seniors are allowed to leave campus for lunch. Sketch a graph of the number of cars in the student parking lot from 6 am to 4 pm.

3. A train that takes passengers from downtown back home to the suburbs makes 5 stops. The maximum speed at which the train can travel is 40 mph. Sketch a graph of the speed of the train a function of time since leaving the downtown train station.

4. Sketch the graph of the total number of people that have seen the hit movie of the summer as a function of the time since opening day of the movie.

4.3f Self-Assessment: Section 4.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Deep Understanding, Skill Mastery
1. Determine whether the relationship between two quantities can be modeled by a linear function.			
2. Construct a function to model a linear relationship between two quantities.			
3. Identify and interpret key features of a graph that models a relationship between two quantities.			
4. Sketch a graph that displays key features of a function that has been described verbally.			