Math III Exponential/Geometric Series

Exponential functions are of the form \( y = a(b)^x \), where \( a \) is the y-intercept or initial amount, and \( b \) is the growth/decay factor. If \( b > 1 \) it represents exponential growth and if \( 0 < b < 1 \) it represents exponential decay.

\[
\text{Ex1: } f(x) = 5,236(1.08)^x \text{ exponential growth, growth rate is 8%} \\
\text{Ex2: } f(x) = 2,873(0.91)^x \text{ exponential decay, decay rate is 9%}
\]

**Compounded interest** uses the formula \( A = p\left(1 + \frac{r}{n}\right)^{nt} \), where \( p \) is the principle, \( r \) is the rate, \( t \) is the time, and \( n \) is the number of times the interest is compounded. (monthly \( n = 12 \), weekly \( n = 52 \), etc.)

**Continuously compounded interest** uses the formula \( A = Pe^{rt} \), where \( p \) is the principle, \( r \) is the rate, and \( t \) is the time.

**Mortgage Formula** - monthly payment \( = \frac{pi}{1-(1+i)^{-n}} \) where \( p \) is the principle, \( n \) is the number of total payments, \( i \) is the monthly interest rate.

**Sum of finite geometric series** is found by \( S_n = \frac{a_1(1-r^n)}{1-r} \), where \( a_1 \) is the first term, \( r \) is the ratio (what each term is multiplied by to get to the next), \( n \) is term number the series stops.

Examples:

1. 288, -96, 32, ... What is the approximate value of the sum of the 7th term?
   \[
   \frac{288(1-(-\frac{1}{3})^7)}{1-(-\frac{1}{3})} = 216.1
   \]

2. 360 + 480 + 640 + ... What is the approximate value of the sum of the 15th term?
   \[
   \frac{360(1-\left(\frac{4}{3}\right)^{15})}{1-\frac{4}{3}} = 791.737.39
   \]

3. What is the approximate value of the sum:
   \[
   8 - \frac{8}{7} + \frac{8}{49} - \cdots 8 \cdot \left(\frac{-1}{7}\right)^{2500} \quad \text{or} \quad \frac{8(1-(-\frac{1}{7})^{2500})}{1-(-\frac{1}{7})} = 7
   \]
4. Find the monthly payment of $175,000 home on a 30 year mortgage with a 3.5% interest rate.

\[
i = 1.035^{\frac{1}{12}} = 1.00287
\]

\[
175,000 \times \frac{0.00287}{1 - (1 + 0.00287)^{-360}} = 780.37
\]

5. Angela deposited $3000 into a savings account earning 4% interest compounded continuously, how much will she have after 6 years? 

\[
P e^{rt} = 3813.75
\]

6. Sam deposited $5,500 into a savings account earning 5.6% interest compounded monthly. How many years had he been saving when the savings account has a balance of $8599.52?

\[
8,599.52 = 5,500 \times (1 + \frac{0.056}{12})^{12x}
\]

\[
\frac{8,599.52}{5,500} = (1.0046)^{12x}
\]

\[
1.56 = (1.0046)^{12x}
\]

\[
\log_{1.0046} 1.56 = 12x
\]

\[
x = 8.07
\]

7. Mary wants a dress that costs $450 for the prom. So far she has saved $275 and put it in a savings account for 1.5 years, what interest rate must she earn to have $450 by prom? (compounded continuously)

\[
\frac{450}{275} = e^{1.5x}
\]

\[
\ln 1.64 = 1.5x
\]

\[
x = 0.329 = 32.9\%
\]

8. A board is made up of 9 squares. A certain number of pennies is placed in each square, following a geometric sequence. The first square has 1 penny, the second has 2 pennies, the third has 4 pennies, etc. When every square is filled, how many pennies will be used in total?

A. 521  B. 511  C. 256  D. 81

\[
\begin{array}{ccc}
1 & 2 & 4 \\
8 & 16 & 32 \\
64 & 128 & 256 \\
\end{array}
\]