Unit 1
Resource Masters
Patterns of Change

Christian R. Hirsch • James T. Fey • Eric W. Hart
Harold L. Schoen • Ann E. Watkins

with
Beth E. Ritsema • Rebecca K. Walker • Sabrina Keller
Robin Marcus • Arthur F. Coxford • Gail Burrill
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Core-Plus Mathematics
Using the Unit Resource Masters

Overview of Unit Resource Masters

To assist you as you teach Course 1 of Core-Plus Mathematics, this unit-specific resource book has been developed. The unit resources provided can help you focus student attention on the important mathematics being developed. They can be used to help students organize their results related to specific problems, synthesize what they are learning, and practice for standardized tests.

Each unit resource book provides the following masters in the order that they are used in the unit.

- **Transparency Masters**
  1. Think About This Situation (TATS) masters to help launch the lesson
  2. Masters to collect class results
  3. Summarize the Mathematics (STM) masters to help facilitate the synthesis of mathematical ideas from the investigation (To guide your planning, sample discussion scenarios called “Promoting Mathematical Discourse” are provided in the Teacher’s Guide for selected TATS and STM discussions.)

- **Student Masters**
  1. Masters to help students organize their results
  2. Technology Tips to facilitate learning technology features of graphing calculators, spreadsheet software, and computer algebra systems (CAS)
  3. Unit Summary masters to provide a starting point for pulling together the main mathematical ideas of a unit
  4. Practicing for Standardized Tests masters provide an opportunity for students to complete tasks presented in the format of most high-stakes tests and to consider test-taking strategies. (Solutions to these tasks are printed in the Teacher’s Guide following the final unit Summarize the Mathematics. This allows you the option of providing or not providing the solutions to students.)

- **Assessment Masters**
  1. Quizzes (two forms for each lesson)
  2. In-class tests (two forms for each unit)
  3. Take-home assessment items (three items for each unit)
  4. Projects (two or three for each unit)
  5. Midterm and end-of-course assessment items (Unit 4 and Unit 8 contain a bank of assessment items from which to design cumulative exams.)

All of the items in this book are included for viewing and printing from the Core-Plus Mathematics TeacherWorks Plus CD-ROM. Custom tailoring of assessment items in this book, as well as creation of additional items, can be accomplished by using the ExamView Assessment Suite.
Assessment in Core-Plus Mathematics

Throughout the Core-Plus Mathematics curriculum, the term “assessment” is meant to include all instances of gathering information about students’ levels of understanding and their disposition toward mathematics for purposes of making decisions about instruction. The dimensions of student performance that are assessed in this curriculum (see chart below) are consistent with the assessment recommendations of the National Council of Teachers of Mathematics in the Assessment Standards for School Mathematics (NCTM, 1995). They are more comprehensive than those of a typical testing program.

<table>
<thead>
<tr>
<th>Assessment Dimensions</th>
<th>Content</th>
<th>Disposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Problem Solving</td>
<td>Concepts</td>
<td>Beliefs</td>
</tr>
<tr>
<td>Reasoning</td>
<td>Applications</td>
<td>Perseverance</td>
</tr>
<tr>
<td>Communication</td>
<td>Representational Strategies</td>
<td>Confidence</td>
</tr>
<tr>
<td>Connections</td>
<td>Procedures</td>
<td>Enthusiasm</td>
</tr>
</tbody>
</table>

These unit resource masters contain the tools for formal assessment of the process and content dimensions of student performance. Calculators are assumed in most cases on these assessments. Teacher discretion should be used regarding student access to their textbooks and Math Toolkits for assessments. In general, if the goals to be assessed are problem solving and reasoning, while memory of facts and procedural skill are of less interest, resources should be allowed. However, if automaticity of procedures or unaided recall are being assessed, it is appropriate to prohibit resource materials.

You may want to consult the extended section on assessment in the front matter of the Course 1 Core-Plus Mathematics Teacher’s Guide and Implementing Core-Plus Mathematics. Among the topics presented in these sources are curriculum-embedded assessment, student-generated assessment, and scoring assessments and assigning grades. Since the Core-Plus Mathematics approach and materials provide a wide variety of assessment information, the teacher will be in a good position to assign grades. With such a wide choice of assessment opportunities, a word of caution is appropriate: It is easy to overassess students, and care must be taken to avoid doing so. Since many rich opportunities for assessing students are embedded in the curriculum itself, you may choose not to use a quiz at the end of every lesson or to replace all or portions of an in-class test with take-home tasks or projects.
Collaborative Group Guidelines

- Each member contributes to the group’s work.
- Each member of the group is responsible for listening carefully when another group member is talking.
- Each member of the group has the responsibility and the right to ask questions.
- Each group member should help others in the group when asked.
- Each member of the group should be considerate and encouraging.
- All members should work together until everyone in the group understands and can explain the group’s results.
## Evaluating My Collaborative Work

<table>
<thead>
<tr>
<th></th>
<th>Yes</th>
<th>Somewhat</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I participated in this investigation by contributing ideas.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. I was considerate of others, showed appreciation of ideas, and encouraged others to respond.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. I paraphrased others’ responses and asked others to explain their thinking and work.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. I listened carefully and disagreed in an agreeable manner.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. I checked others’ understanding of the work.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. I helped others in the group understand the solution(s) and strategies.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. We all agreed on the solution(s).</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. I stayed on task and got the group back to work when necessary.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. We asked the teacher for assistance only if everyone in the group had the same question.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. What actions helped the group work productively?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. What actions could make the group even more productive tomorrow?</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Your signature: ___________________________
Constructing a Math Toolkit
Suppose that operators of Five Star Amusement Park are considering installation of a bungee jump.

**a** How could they design and operate the bungee jump attraction so that people of different weights could have safe but exciting jumps?

**b** Suppose one test with a 50-pound jumper stretched a 60-foot bungee cord to a length of 70 feet. What patterns would you expect in a table or graph showing the stretched length of the 60-foot bungee cord for jumpers of different weights?

<table>
<thead>
<tr>
<th>Jumper Weight (in pounds)</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stretched Cord Length (in feet)</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**c** How could the Five Star Amusement Park find the price to charge each customer so that daily income from the bungee jump attraction is maximized?

**d** What other safety and business problems would Five Star Amusement Park have to consider in order to set up and operate the bungee attraction safely and profitably?
Bungee Jump Experiment

<table>
<thead>
<tr>
<th>Weight Attached</th>
<th>Length of Stretched Cord</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Summarize the Mathematics

To describe relationships among variables, it is often helpful to explain how one variable is a function of the other or how the value of one variable depends on the value of the other.

a How would you describe the way that:
   
i. the stretch of a bungee cord depends on the weight of the jumper?
   
   ii. the number of customers for a bungee jump attraction depends on the price per customer?
   
   iii. income from the jump depends on price per customer?

b What similarities and what differences do you see in the relationships of variables in the physics and business questions about bungee jumping at Five Star Amusement Park?

c In a problem situation involving two related variables, how do you decide which should be considered the independent variable? The dependent variable?

d What are the advantages and disadvantages of using tables, graphs, algebraic rules, or descriptions in words to express the way variables are related?

e In this investigation, you were asked to use patterns in data plots and algebraic rules to make predictions of bungee jump stretch, numbers of customers, and income. How much confidence or concern would you have about the accuracy of those predictions?

Be prepared to share your thinking with the whole class.
## Take a Chance

### Problems 1 and 2

<table>
<thead>
<tr>
<th>Play Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome ($ won or lost for school)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cumulative Profit ($ won or lost by school)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Play Number</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome ($ won or lost for school)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cumulative Profit ($ won or lost by school)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Play Number</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome ($ won or lost for school)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cumulative Profit ($ won or lost by school)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Play Number</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome ($ won or lost for school)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cumulative Profit ($ won or lost by school)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Graph](image-url)
Take a Chance
Problems 3 and 4

$4 Prize Payoff

<table>
<thead>
<tr>
<th>Number of Plays</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
<th>160</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative Profit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$6 Prize Payoff

<table>
<thead>
<tr>
<th>Number of Plays</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
<th>160</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative Profit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Summarize the Mathematics**

In this investigation, you explored patterns of change for a variable with outcomes subject to the laws of probability. You probably discovered in the die-tossing game that *cumulative profit* is related somewhat predictably to the *number of plays* of the game.

**a** After many plays of the two games with payoffs of $4 or $6, who seemed to come out ahead in the long run—the players or the school fund-raiser? Why do you think those results occurred?

**b** How is the pattern of change in *cumulative profit* for the school fund-raiser similar to, or different from, patterns you discovered in the investigation of bungee physics and business?

*Be prepared to share your ideas and reasoning with the class.*
a. Average Speed (in mph) | 50 | 75 | 100 | 125 | 150 | 175 | 200
--- | --- | --- | --- | --- | --- | --- | ---
Race Time (in hours) |   |   |   |   |   |   |   

b. Symbolic rule:

Specific example checks:

<table>
<thead>
<tr>
<th>Average Speed (in mph)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Race Time (in hours)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Part-Time Work ... Big-Time Dollars

### Problems 5 and 6

<table>
<thead>
<tr>
<th>Hours Worked in a Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Earnings in $ Plan 1</strong></td>
<td>2</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Earnings in $ Plan 2</strong></td>
<td>0.10</td>
<td>0.30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Earnings in $ Plan 3</strong></td>
<td>2</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Earnings in $ Plan 4</strong></td>
<td>0.10</td>
<td>0.30</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hours Worked in a Week</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Earnings in $ Plan 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Earnings in $ Plan 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Earnings in $ Plan 3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Earnings in $ Plan 4</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The patterns relating *race time* to *average speed* for the Daytona 500 and *earnings* to *hours worked* in Plan 2 at Fresh Fare Market are examples of nonlinear relationships.

a) What is it about those relationships that makes the term “nonlinear” appropriate?

b) You found patterns showing how to calculate *race time* from *average speed* and *total pay* from *hours worked*. How would your confidence about the accuracy of those calculations compare to that for calculations in the bungee jump and fair game problems?

*Be prepared to share your ideas and reasoning with the class.*
Lesson 1 Quiz

Form A

1. The Debate Team at your school is selling cookies as a fund-raiser. You need to decide how much to charge for each cookie. You take a poll and estimate the total number of cookies that you can sell at different prices. The results are provided in the table below.

<table>
<thead>
<tr>
<th>Price per Cookie (in cents)</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>60</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Cookies Sold</td>
<td>600</td>
<td>500</td>
<td>400</td>
<td>200</td>
<td>50</td>
</tr>
</tbody>
</table>

a. In this situation, which variable is naturally independent and which is dependent? Explain your reasoning.

Independent: ____________________________  Dependent: ____________________________
Explanation: ____________________________

b. Plot the given data on the coordinate grid provided below.

![Coordinate Grid]

c. Use the pattern in the table or graph to estimate the number of cookies you will sell if you sell them for 35¢ each. For 70¢ each. Explain your reasoning.

35¢ each: ____________________________  70¢ each: ____________________________
Explanation: ____________________________
d. The local bakery will donate 300 cookies for your sale. What should you charge per cookie so that you sell them all? Explain your reasoning.

*Cost per Cookie: ____________*

*Explanation:*


e. Describe as precisely as possible the overall pattern of change relating price per cookie and number of cookies sold.

*Description:*

2. Pat competes in the 1,600-meter run for his high school track team. Clearly the time it takes Pat to complete the run depends on his average running speed. If Pat’s average speed is 2 meters per second, it will take him 800 seconds to complete the race.

a. Complete the table below showing the way that race time and average speed are related.

<table>
<thead>
<tr>
<th>Average Speed (in meters per second)</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Race Time (in second)</td>
<td>800</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. On the grid below, make a graph that shows how race time changes as average speed increases. Be sure to properly label your graph.

---

**Description:**

b. On the grid below, make a graph that shows how race time changes as average speed increases. Be sure to properly label your graph.

**Description:**
1. a. Independent: Price per cookie  
Dependent: Number of cookies sold  
The price per cookie is independent since it is what the students have control over. Once they  
pick a price, the number of cookies they will sell is determined.

b. 450 cookies at 35¢ each and 100 cookies at 70¢ each
Using the table: Since 35¢ is halfway between 30¢ and 40¢, it seems reasonable that the number  
of cookies sold would be halfway between 500 and 400 or 450. From the table, you can also see  
that a 10¢ increase in price is associated with a 100-cookie decrease in sales. So, since 70¢ is 10¢  
more than 60¢, 200 – 100 or 100 cookies will be sold.

Using the graph: Students could connect the dots with a line and then find the appropriate price  
on the x-axis, read up to the line and then over to the y-axis to find the number of cookies that  
will be sold.
d. 50¢. Since 300 is halfway between 200 and 400, you should set the price halfway between 40¢ and 60¢, or at 50¢.

e. As the price goes up by 10¢, the number of cookies sold goes down by 100.

<table>
<thead>
<tr>
<th>Average Speed (in meters per second)</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate Time (in seconds)</td>
<td>800</td>
<td>400</td>
<td>266.67</td>
<td>200</td>
<td>160</td>
</tr>
</tbody>
</table>

2. a. As average speed increases, the race time decreases. The race time decreases rapidly at first and then more slowly.
1. The graph below shows the stretched length of a spring with different weights attached to it.

![Graph of stretched length vs. weight](image)

**a.** Estimate the stretched length of the spring if a 2-pound weight is attached. Then estimate the stretched length of a spring if a 5-pound weight is attached. Explain how you made your estimate.

2-pound weight: ____________  
5-pound weight: ____________

Explanation:

**b.** If the stretched length of the spring is 10 feet, estimate how much weight is attached to the spring. Explain how you found your answer.

Weight: ____________

Explanation:
c. Trisha thought that a formula could be used to help her make a more accurate prediction. She let $L$ represent the stretched length and $w$ the weight and suggested using the rule $L = 1 + 1.5w$. Will her rule produce weight and length pairs close to those indicated by the graph? Explain your reasoning.

*Explanation:*

d. Explain as clearly as possible the pattern relating attached weight and stretched length of the spring.

*Explanation:*

2. Your grandparents need to have some work done around their house. They are willing to pay you $200 to do the job. You decide to get some of your friends to help you with the job, and you agree that you will split the $200 equally among all the people who work. So, if 2 people (you and a friend) work, each of you will make $100.

a. Complete the table below showing the way that pay per person and total number of people are related.

<table>
<thead>
<tr>
<th>Total Number of People</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pay per Person</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. In this situation, which variable is naturally independent and which is naturally dependent? Explain your reasoning.

*Independent: ________________  Dependent: ________________

*Explanation: *
c. On the grid below, make a graph that shows how pay per person changes as total number of people increases. Be sure to finish labeling the graph.

\[ 
\begin{array}{c}
\text{Total Number of People} \\
0 & 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18 & 20 & 22 & 24 \\
\end{array} 
\]

\[ 
\begin{array}{c}
\text{Pay per Person} \\
\end{array} 
\]

d. Describe the pattern of change shown in the graph above.

*Description:*

e. Which increase in total number of people will result in a greater decrease in pay per person: an increase from 5 to 10 people or an increase from 10 to 15 people? How is this shown on the graph?
1. a. 2-pound weight: 4 feet
   5-pound weight: 8.5 feet
   To find the stretched length for a given weight, first locate the weight on the horizontal axis. Then move vertically until you intersect with the line. Look horizontally over to the y-axis to identify the y-coordinate of the point on the line; that value is the stretched length of the spring for the given weight.

b. 6 pounds is attached to the spring. Move horizontally from the 10 on the y-axis across to the line. Then from the point of intersection, look vertically down to identify the x-coordinate of the point. That is the weight that is associated with a stretched length of 10 feet.

c. This rule will produce the same (weight, stretched length) pairs as indicated by the line.

d. As more weight is attached, the stretched length increases. Specifically, for each 1 pound of weight that is added to the spring, the stretched length increases by 1.5 feet.

2. a. | Total Number of People | 1   | 2   | 5   | 10  | 20  |
     | Pay per Person          | 200 | 100 | 40  | 20  | 10  |

b. Independent: Total number of people
   Dependent: Pay per person
   Since the money is divided equally among the workers, the total number of people who work determines how much each will get paid. So we can say that pay per person depends on total number of people.
c. Students may draw either a scatterplot or a continuous curve.

\[ \text{Pay per Person (in dollars)} \]

\[ \text{Total Number of People} \]

---

d. The pay per person decreases as the total number of people working increases. It decreases rapidly at first and then more slowly.

e. An increase from 5 to 10 people produces a $20 per person decrease in pay and an increase from 10 to 15 people produces a $6.67 per person decrease in pay. Thus, the increase for 5 to 10 people results in a greater pay decrease. This is shown on the graph because the graph is steeper (drops more quickly) between 5 and 10 than it is between 10 and 15.
Think About This Situation

The population of the world and of individual countries, states, and cities changes over time.

a How would you describe the pattern of change in world population from 1650 to 2050?

b What do you think are some of the major factors that influence population change of a city, a region, or a country?

c How could governments estimate year-to-year population changes without making a complete census?
Population Change in Brazil

Problem 2

• Based on recent trends, births every year equal about 1.7% of the total population of the country.

• Deaths every year equal about 0.6% of the total population.

Population Estimates for Brazil

<table>
<thead>
<tr>
<th>Year</th>
<th>Change (in millions)</th>
<th>Total Population (in millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>—</td>
<td>186</td>
</tr>
<tr>
<td>2006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2007</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In the studies of human and whale populations, you made estimates for several years based on growth trends from the past.

**a** What trend data and calculations were required to make these estimates:

1. The change in the population of Brazil from one year to the next? The new total population of that country?
2. The change in number of Alaskan bowhead whales from one year to the next? The new total whale population?

**b** What does a NOW-NEXT rule like $NEXT = 1.03 \cdot NOW - 100$ tell about patterns of change in a variable over time?

**c** What calculator commands can be used to make population predictions for many years in the future? How do those commands implement NOW-NEXT rules?

*Be prepared to share your thinking with the class.*
Spreadsheet Work

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Name ____________________________
Date _____________________________
Learning to Use Spreadsheets

To develop or refresh your memory of the basic skills required in spreadsheet design, work through the following steps in constructing and testing a spreadsheet similar to the growth of Alaskan bowhead whale population example on pages 32–33.

a. Start by typing the information shown below in cells A1:A2, cells B1:B2, and cells C1:C4. To type words or numbers in a cell, click on that cell, type the desired entry, and press the “enter” key. Cells are labeled with a letter indicating the column and a number indicating the row. For example, cell C2 is at the intersection of the third column and the second row. To widen a column, move the cursor to the right of the lettered column heading until the column divider cursor appears. Then slide the divider to the right until the column widens to the desired width.

To change a cell entry, click on that cell, delete the contents of the cell, type the new information, and press “enter.”

b. Next, to display the formulas that will generate year numbers 2002–2005 type the formula “=A2+1” in cell A3 and press “enter.” Then, highlight cells A3:A6 and use the “fill down” command in the Edit menu. You should get a result like this:
c. Now enter the formulas for calculating the population in every year. You need to show the pattern in cell B3 and then use the fill down command in the Edit menu to repeat the pattern in cells below. The first step of this spreadsheet extension should look like this:

![Spreadsheet image]

After using the fill down command, you should get a result like this:

![Spreadsheet image]

In Microsoft Excel, you can adjust the number of decimal places displayed by selecting the cells and looking under Format-Cells-Number. Other spreadsheet software should also have this feature.

d. To utilize fixed values like “Natural Growth Rate” and “Hunting Rate,” use $ symbols before C and 2 and before C and 4 to keep those cell references fixed in the fill down operation.

![Spreadsheet image]

After using the fill down command in the Edit menu, you should have the same result as you did in Part c.

e. Experiment with the spreadsheet by changing growth and hunting rates to investigate the power of using the $ in fixed cell references.
In this investigation, you learned basic spreadsheet techniques for studying patterns of change.

a) How are cells in a spreadsheet grid labeled and referenced by formulas?

b) How are formulas used in spreadsheets to produce numbers from data in other cells?

c) How is the “fill” command used to produce cell formulas rapidly?

d) How are the cell formulas in a spreadsheet similar to the NOW-NEXT rules you used to predict population change?

*Be prepared to share your ideas with other students.*
City Reservoir Water Amounts

Percent of Reservoir Capacity

Time (in days)
LESSON 2 QUIZ

Form A

1. To encourage you to save, your parents open a savings account in your name. This savings account earns 1.5% per month interest on the balance at the end of each month. Suppose that your grandparents deposit $2,000 into the account on January 1.
   
a. If you do not take any money out of, or put any more money into, the account, how much money will be in the account at the end of one month? Two months? Explain or show your work.
   
   End of one month: _____________  
   
   End of two months: _____________  
   
   Show your work:

   b. When will you have $2,500? Explain how you got your answer.

   The account reaches $2,500 after _________________

   Explanation:

   c. Using NOW to stand for the amount in the account at the end of one month, write a rule for calculating the amount in the account at the end of the NEXT month. Explain how your rule reflects the given situation.

   NEXT = _________________ , starting at _________________

   Explanation:

   d. Suppose you take out $200 at the end of each month starting in January. How much will you have in the account at the end of January? Of February? Show or explain your work.

   End of January: _________________  
   
   End of February: _________________  

   Explanation:

   e. Assume the conditions described in Part d. Using NOW to stand for the amount you have in the account at the end of one month, write a rule for calculating the amount you have left in the account at the end of the NEXT month.

   NEXT = _________________ , starting at _________________
f. Under the conditions described in Part d and assuming there are no deposits into the account, after how many months will the account have no money left in it? Show or explain your work.

Account balance will reach zero after _________________ months.

Work or explanation:

2. The spreadsheet below is one that can be used to explore the situation described in Problem 1, Part d above.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Months Passed</td>
<td>Account Balance</td>
<td>Interest Rate</td>
</tr>
<tr>
<td>0</td>
<td>2,000</td>
<td>0.015</td>
</tr>
<tr>
<td>Monthly Withdrawal</td>
<td>200</td>
<td></td>
</tr>
</tbody>
</table>

a. What formula could you place in cell A3 so that you could then use the fill down command to complete the rest of the column?

b. What formula could you place in cell B3 so that you could then use the fill down command to complete the rest of the column?

3. For each rule below, produce a table of values showing how the quantity changes from the start through four stages of change.

a. \( \text{NEXT} = \text{NOW} - 0.25 \times \text{NOW} \), starting at 160

<table>
<thead>
<tr>
<th>Stage</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

b. \( \text{NEXT} = \text{NOW} + 0.5 \times \text{NOW} + 5 \), starting at 10

<table>
<thead>
<tr>
<th>Stage</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
Lesson 2 Quiz

Form A
Suggested Solutions

1. a. One month: \(2,000 \cdot 1.015 = \$2,030\)
   Two months: \(2,030 \cdot 1.015 = \$2,060.45\)

b. The balance reaches \$2,500 after 15 months. To determine this, continue to multiply the amount in the account by 1.015 until the balance first reaches \$2,500 and then count the number of times you had to multiply by 1.015.

c. \(NEXT = 1.015 \cdot NOW\), starting at 2,000
   The initial deposit is \$2,000 so that is the starting point. The interest earned each month is 1.5%, so each time you must multiply by 1.015.

d. January: \(2,000(1.015) - 200 = \$1,830\)
   February: \(1,830(1.015) - 200 = \$1,657.45\)
   First multiply the previous month’s balance by 1.015, then subtract 200.

e. \(NEXT = 1.015 \cdot NOW - 200\), starting at 2,000

f. 11 months. You must multiply by 1.015 and subtract 200 eleven times before the amount in the account is less than zero. The withdrawal in the eleventh month will only be \$180.54.

2. a. \(A3 = A2 + 1\)

b. \(B3 = B2 \cdot (1 + $C2) - C4\) or \(B3 = B2 \cdot 1.015 - 200\)

3. a. 

<table>
<thead>
<tr>
<th>Stage</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>160</td>
<td>120</td>
<td>90</td>
<td>67.5</td>
<td>50.625</td>
</tr>
</tbody>
</table>

b. 

<table>
<thead>
<tr>
<th>Stage</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>10</td>
<td>20</td>
<td>35</td>
<td>57.5</td>
<td>91.25</td>
</tr>
</tbody>
</table>
1. The annual change in the population of Egypt depends on the population the previous year, the number of people born each year, the number of people who die each year and the number of people who move to or leave Egypt each year. These statistics for Egypt are given below.

- Births every year will equal about 2.3% of the total population.
- Deaths every year will equal about 0.5% of the population.
- Every year approximately 0.02 million more people will leave Egypt than will move to Egypt.
- The 2005 population of Egypt was 77.5 million.

**Source:** CIA – The World Factbook 2005


<table>
<thead>
<tr>
<th>Year</th>
<th>Population (in Millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td></td>
</tr>
<tr>
<td>2007</td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td></td>
</tr>
</tbody>
</table>

b. Describe how you made your estimates in Part a.

*Description:*

c. Use the words NOW and NEXT to write a rule that matches your description in Part b.

*Rule:*

d. When will the population first reach 90 million people?
e. Describe the pattern of change over time if the births every year decreased to 0.5% and everything else stayed the same.

*Description:*

2. Use the words NOW and NEXT to write rules that match each table below. Explain your reasoning.

a. 

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
</tr>
</tbody>
</table>

NEXT = ______________, starting at ______________

b. 

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>5</td>
<td>10</td>
<td>20</td>
<td>40</td>
<td>80</td>
</tr>
</tbody>
</table>

NEXT = ______________, starting at ______________

3. Consider the beginning of the spreadsheet below.

```
A   B
1   1  3
2   2 14
3
4
5
6
```

a. Suppose the formula “=A2+1” was placed in cell A3, and a fill down command was used to create the rest of the column. Put the correct numbers in cells A3 through A6.

b. Suppose the formula “=2*B2+5” was placed in cell B3, and a fill down command was used to create the rest of the column. Put the correct numbers in cells B3 through B6.
1. a. The completed spreadsheet is given below.

<table>
<thead>
<tr>
<th>Year</th>
<th>Population (in millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>78.88</td>
</tr>
<tr>
<td>2007</td>
<td>80.27</td>
</tr>
<tr>
<td>2008</td>
<td>81.70</td>
</tr>
<tr>
<td>2009</td>
<td>83.15</td>
</tr>
</tbody>
</table>

b. The change due to births and deaths combined is $2.3\% - 0.5\% = 1.8\%$, which is equal to 0.018. Thus, you need to multiply the population in a given year by 1.018 and then subtract 0.02 (net migration) to get the population for the following year.

c. $\text{NEXT} = 1.018 \cdot \text{NOW} - 0.02$, starting at 77.5 million

d. The population in 2013 is predicted to be 89.21 million and in 2014 it is predicted to be 90.80 million. So, during 2013 the population will first reach 90 million.

e. This would mean that the only change in population would be due to people moving in and out of the country. That means that the population would decrease by 0.02 million people every year. The change would be the same every year.

2. a. $\text{NEXT} = \text{NOW} + 5$, starting at 10
    
    b. $\text{NEXT} = 2 \cdot \text{NOW}$, starting at 5

3. a–b. The completed spreadsheet is given below.
Think About This Situation

If you were asked to solve problems in situations similar to those described below:

a How would you go about finding algebraic rules to model the relationships between dependent and independent variables in any particular case?

b What ideas do you have about how the forms of algebraic rules are connected to patterns in the tables and graphs of the relationships that they produce?

c How could you use calculator or computer tools to answer questions about the variables and relationships expressed in rules?

The stretched length $L$ of a simulated bungee cord depends on the attached weight $w$ in a way that is expressed by the formula

$$L = 30 + 0.5w.$$ 

The number of customers $n$ for a bungee jump depends on the price per jump $p$ in a way that is expressed by the rule $n = 50 - p$.

The time $t$ of a 500-mile NASCAR race depends on the average speed $s$ of the winning car in a way that is expressed by the rule $t = \frac{500}{s}$. 
In this investigation, you developed your skill in finding symbolic rules for patterns that relate dependent and independent variables.

a. What strategies for finding algebraic rules do you find helpful when information about the pattern comes in the form of words describing the relationship of the variables?

b. In general, what information is needed to calculate perimeter and area for:
   
   i. a rectangle?
   
   ii. a parallelogram that is not a rectangle?
   
   iii. a right triangle?
   
   iv. a non-right triangle?
   
   v. a circle?

C. What formulas guide calculations of perimeter and area for each figure listed in Part b?

Be prepared to share your strategies and results with the class.
# Producing Tables of Function Values

<table>
<thead>
<tr>
<th>Calculator Commands</th>
<th>Expected Display</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Enter Formula:</strong></td>
<td><img src="image1" alt="Expected Display" /></td>
</tr>
<tr>
<td>$Y = X,T,\theta,\phi \left( 50 - X,T,\theta,\phi \right)$</td>
<td></td>
</tr>
</tbody>
</table>

**Set Up Table:**
- 2nd WINDOW
- Table Start: 0
- Table Steps: 10

**Display Graph:**
- 2nd GRAPH
### Producing Graphs of Functions

<table>
<thead>
<tr>
<th>Calculator Commands</th>
<th>Expected Display</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Enter Formula:</strong></td>
<td><img src="image1" alt="Graph with equation" /></td>
</tr>
<tr>
<td><img src="image2" alt="Y=" /> X,T,θ,n <img src="image3" alt="50" /> <img src="image4" alt="X,T,θ,n" /></td>
<td></td>
</tr>
</tbody>
</table>

**Set Viewing Window:**

```
WINDOW (-) 10 ENTER 60 ENTER etc.
```

**Display Graph:**

![GRAPH](image5)

**Trace the Graph:**

![TRACE](image6)
## Solving an Equation

<table>
<thead>
<tr>
<th>Calculator Commands</th>
<th>Expected Display</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>F2</strong></td>
<td><img src="image1.png" alt="TI-89 Screen 1" /></td>
</tr>
<tr>
<td>ENTER ALPHA STO × (50 - )</td>
<td><img src="image2.png" alt="TI-89 Screen 2" /></td>
</tr>
<tr>
<td>ALPHA STO ÷ 1 = 500 ÷ ALPHA STO ÷ 1</td>
<td><img src="image3.png" alt="TI-89 Screen 3" /></td>
</tr>
<tr>
<td>ENTER</td>
<td><img src="image4.png" alt="TI-89 Screen 4" /></td>
</tr>
</tbody>
</table>

To obtain approximate answers without changing from auto mode to approximate mode, arrow over on the equation line and insert a decimal point after 50.
# Evaluating an Expression

<table>
<thead>
<tr>
<th>Calculator Commands</th>
<th>Expected Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALPHA STO ➙ x ( 50 ➖ ALPHA STO ➙ ) ➗ 5 ➙ x ( 2nd ➙ x 5 ➙ ) ➗ 5 ➙ )</td>
<td><img src="image" alt="TI-89 Calculator" /></td>
</tr>
</tbody>
</table>

**TI-89 Calculator Commands Expected Display**

- **TI-89 Calculator Commands**: Use the following commands to evaluate an expression on a TI-89 calculator:
  - **ALPHA**: Press the ALPHA button to access the menu of functions.
  - **STO ➙**: Store values or expressions.
  - **═**: Equal sign to calculate the expression.
  - **2nd ➙ x**: Access 2nd function for x, enabling more advanced operations.

- **Expected Display**: The display should show the evaluated expression and result. For example, if evaluating an expression like `(50 ➖ ALPHA ➙ ➗ 5) ➗ 5 ➙ )`, the display should show the evaluated expression and the result. The image shows a sample TI-89 calculator display with the evaluated expression and result.
In this investigation, you developed skill in use of calculator or computer tools to study relations between variables. You learned how to construct tables and graphs of pairs of values and how to use a computer algebra system to solve equations.

**a** Suppose that you were given the algebraic rule \( y = 5x + \frac{10}{x} \) relating two variables. How could you use that rule to find:

- the value of \( y \) when \( x = 4 \)
- the value(s) of \( x \) that give \( y = 15 \)

  i. using a table of \((x, y)\) values?

  ii. using a graph of \((x, y)\) values?

  iii. using a computer algebra system?

**b** What seem to be the strengths and limitations of each tool—table, graph, and computer algebra system—in answering questions about related variables? What do these tools offer that makes problem solving easier than it would be without them?

*Be prepared to share your thinking with the class.*
Explorations

<table>
<thead>
<tr>
<th>$x$</th>
<th>−5</th>
<th>−4</th>
<th>−3</th>
<th>−2</th>
<th>−1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Rule:

Window:

$X_{\text{min}} = −5$
$X_{\text{max}} = 5$
$X_{\text{scl}} = 1$
$Y_{\text{min}} = $
$Y_{\text{max}} =$
$Y_{\text{scl}} =$

<table>
<thead>
<tr>
<th>$x$</th>
<th>−5</th>
<th>−4</th>
<th>−3</th>
<th>−2</th>
<th>−1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Rule:

Window:

$X_{\text{min}} = −5$
$X_{\text{max}} = 5$
$X_{\text{scl}} = 1$
$Y_{\text{min}} = $
$Y_{\text{max}} =$
$Y_{\text{scl}} =$
Summarize the Mathematics

As a result of the explorations, you probably have some ideas about the patterns in tables of \((x, y)\) values and the shapes of graphs that can be expected for various symbolic rules. Summarize your conjectures in statements like these:

\(\textbf{a}\) If we see a rule like \(\ldots\), we expect to get a table like \(\ldots\) .

\(\textbf{b}\) If we see a rule like \(\ldots\), we expect to get a graph like \(\ldots\) .

\(\textbf{c}\) If we see a graph pattern like \(\ldots\), we expect to get a table like \(\ldots\) .

*Be prepared to share your ideas with others in your class.*
LESSON 3 QUIZ

Form A

1. You have been asked to baby-sit during a local community meeting. You will get paid $10 plus $1.50 extra for each child that you baby-sit.
   a. What will your pay be if you baby-sit 12 children? Show your work.
      Pay: ________________
   b. Write a rule relating your pay $P$, in dollars to the number of children $N$ that you baby-sit.
      Rule: ________________

2. The figure below is a square.

   a. Use a ruler to make measurements needed to estimate the perimeter and area of the square. You should use inches for your measurements. Show your work.
      Perimeter: ________________  Area: ________________
   b. For any square, what is the minimum number of measurements needed to determine the perimeter and area of the square? What are the required measurements?
   c. What formula shows how to calculate perimeter $P$ of a square from the measurements described in Part b?
      $P =$ ________________
d. The formula for the area of a square is \( A = s^2 \) where \( s \) is the length of one side. Circle the graph below that could be a graph of the relationship between the length of a side of a square and the area of the square. Explain how you can determine this without using your calculator.

![Graphs I, II, and III]

**Explanation:**

3. The height, in meters, of a punted football can be found using the rule \( h = -4.9t^2 + 15t + 1 \), where \( t \) is the number of seconds since the football was punted.

   a. Find the height of the ball after 1.75 seconds. Explain how you got your answer or show your work.

   b. Find the maximum height of the football to the nearest tenth of a meter. Sketch the graph of the function. Be sure to label the vertical axis. Explain how the graph shows the maximum height.

   **Maximum height: \______________\**
   
   **Explanation:**

   c. Assume the football is not caught. Find the time (to the nearest tenth of a second) when the football hits the ground. Show or explain your work.

   **Time when football hits the ground: \___________________________\**
   
   **Explanation:**

Name ____________________________  
Date ____________________________
LESSON 3 QUIZ

Form A

Suggested Solutions

1. a. The pay will be $28.
   \[ 10 + 12(1.50) = 28 \]
   b. \( P = 10 + 1.50N \)

2. a. Perimeter: 6 inches
   Area: 2.25 square inches
   b. You only need to know the length of one side of the square.
   c. \( P = 4s \)
   d. Graph II is the correct graph. Since the formula for the area of a square is of the form \( y = ax^2 \), the graph will be a curve, not a line, that indicates the \( y \) values increase as the \( x \) values increase.

3. a. The ball is approximately 12.24 meters high after 1.75 seconds.
   b. Graphs may vary. The maximum height is approximately 12.5 meters. The maximum height can be determined by zooming in on the graph and tracing to the highest point of the graph.
   c. By zooming in on the table of values, you can see that after 3.1 seconds the football is 0.411 meters above the ground and at 3.2 seconds it is predicted to be \(-1.176\) meters in height. This means that to the nearest tenth of a second, the football hits the ground after 3.1 seconds. The same time can be found by zooming in and tracing the graph rather than using the table of values.
UNIT 1  Patterns of Change

LESSON 3 QUIZ

Form B

1. One long-distance telephone company charges a $4.95 monthly fee and then $0.05 per minute of phone use.

   a. How much will the long-distance bill be if you talk for 350 minutes during the month? Show your work.

   b. Write a rule that shows how the total monthly long-distance bill \( B \) depends on the number of minutes used \( N \).

      Rule: ________________

2. The figure below is an equilateral triangle.

   ![Equilateral Triangle]

   a. Using a centimeter ruler, make the necessary measurements and then estimate the perimeter and area of the triangle. Show your work.

      Perimeter: ________________  Area: ________________

   b. The formulas for the area and perimeter of an equilateral triangle are \( A = 0.433s^2 \) and \( P = 3s \) where \( s \) is the length of one side of the triangle. Without using your calculator, identify which graph at the top of the next page is a possible graph for each of these formulas. Explain how you did this without using your calculator.
3. The community theater has done research and found that the rule that relates their income \( I \) to the price \( p \) of a movie ticket to be \( I = p(100 - 8p) \).

   a. Produce a table of values and sketch a graph of the \((\text{price}, \text{income})\) relationship.

<table>
<thead>
<tr>
<th>Price (in dollars)</th>
<th>Income (in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

   Graph of \( A = 0.433s^2 \):
   
   Explanation:

   Graph of \( P = 3s \):
   
   Explanation:
b. What income can be expected if the price is set at $5.50? Explain how you found your answer.

   Income: ____________
   Explanation:

c. What prices will yield income of at least $275? Explain how you determined your answer.

   Prices: ______________
   Explanation:

d. What price will yield maximum income? Explain how you found your answer.

   Prices: ______________
   Explanation:
LESSON 3 QUIZ

Form B

Suggested Solutions

1. a. The bill will be $4.95 + 0.05(350) = $22.45.
   b. $B = 4.95 + 0.05N$

2. a. Perimeter $= 3(3.5) = 10.5$ cm
   Area $\approx \frac{1}{2}(3.5)(3) = 5.25$ cm²
   b. Graph of $A \approx 0.433s^2$ is Graph I. You know it will be a U-shaped curve since the rule has an $s^2$ in it and it must contain $(0, 0)$.
   Graph of $P = 3s$ is Graph IV. You know the graph will be a line since the rule is of the form $y = ax + b$. Since $b = 0$, it must go through $(0, 0)$.

3. a. 

<table>
<thead>
<tr>
<th>Price (in dollars)</th>
<th>Income (in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>168</td>
</tr>
<tr>
<td>4</td>
<td>272</td>
</tr>
<tr>
<td>6</td>
<td>312</td>
</tr>
<tr>
<td>8</td>
<td>288</td>
</tr>
<tr>
<td>10</td>
<td>200</td>
</tr>
<tr>
<td>12</td>
<td>48</td>
</tr>
</tbody>
</table>

b. If the price is $5.50, the income will be $308. This can best be found using either the table or the rule $I = p(100 - 8p)$ and finding $I$ when $p = 5.50$.

c. Prices between $4.09 and $8.41 will yield income of at least $275. This can be found using either the table or the graph.

d. A ticket price of $6.25 will yield the maximum income for the theater. Students can determine this by using either the graph or the table. Students may indicate that ticket prices of $6.23 to $6.27 give the same maximum income of $312.50 due to rounding.
Summarize the Mathematics

When two variables change in relation to each other, the pattern of change often fits one of several common forms. These patterns can be recognized in tables and graphs of \((x, y)\) data, in the rules that show how to calculate values of one variable from given values of the other, and in the conditions of problem situations.

a Sketch at least four graphs showing different patterns relating change in two variables or change in one variable over time. For each graph, write a brief explanation of the pattern shown in the graph and describe a problem situation that involves the pattern.

b Suppose that you develop or discover a rule that shows how a variable \(y\) is a function of another variable \(x\). Describe the different strategies you could use to:

i. Find the value of \(y\) associated with a specific given value of \(x\).

ii. Find the value of \(x\) that gives a specific target value of \(y\).

iii. Describe the way that the value of \(y\) changes as the value of \(x\) increases or decreases.

iv. Find values of \(x\) that give maximum or minimum values of \(y\).

Be prepared to share your ideas and reasoning with the class.
UNIT 1 Patterns of Change

UNIT SUMMARY

In this unit, you developed skill in recognizing patterns of change in variables, in representing those patterns with tables, graphs, and symbolic rules, in describing the patterns of change in words, and in using tables, graphs, and rules to solve problems.

Sketch graphs of four different patterns of change. For each graph:
- Explain in words how the value of $y$ changes as the value of $x$ changes.
- Describe a problem situation in which the pattern of change is likely to occur.
- Give an example of the type of symbolic rule you would expect to represent it.

As the value of $x$ increases, the value of $y$

Problem Situation: _______________________

Rule: ________________________________

As the value of $x$ increases, the value of $y$

Problem Situation: _______________________

Rule: ________________________________

As the value of $x$ increases, the value of $y$

Problem Situation: _______________________

Rule: ________________________________

As the value of $x$ increases, the value of $y$

Problem Situation: _______________________

Rule: ________________________________
Describe two situations in which the pattern of change in a variable over time is represented well by a **NOW-NEXT** rule. For each situation, give an example of a **NOW-NEXT** rule that you would expect to represent the situation.

• 

• 

Explain how you could use an algebraic rule that relates two variables to answer questions that require solving equations or finding maximum or minimum values if the tool of choice is

• A calculator or computer-produced table of values

• A calculator or computer-produced graph

• A calculator or computer algebra program

Write the perimeter and area rules for the following shapes. Explain the meaning of the variables.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>General Parallelogram</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Right Triangle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>General Triangle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Circle</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. The figures below show growth in a pattern of geometric figures.

![Figure 0](image0) ![Figure 1](image1) ![Figure 2](image2) ![Figure 3](image3)

a. Complete the table below to indicate the number of square tiles that would be needed to make each figure.

<table>
<thead>
<tr>
<th>Figure Number $F$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Square Tiles $N$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Think about how the figures change as you move from one figure to the next and look at the table of values in Part a. Write a rule using $NOW$ and $NEXT$ that indicates how the number of tiles changes from one figure to the next.

$NEXT = \underline{\phantom{0000}}$, starting at $\underline{\phantom{0000}}$

Explanation:

c. Circle the rule that indicates how the number of tiles $N$ depends on the figure number $F$.

$N = 7F$ \hspace{1cm} $N = 7 + F$ \hspace{1cm} $N = 7^F$ \hspace{1cm} $N = \frac{7^F}{F}$
2. A nearby lake is very popular for trout fishing. The state wildlife management agency monitors and manages the number of fish in the lake so that there will always be enough trout. The change in the number of trout in the lake is determined by the following.

- There are currently 3,000 trout in the lake.
- The fish population decreases at a rate of 10% per year. This is due to natural causes and fishing combined.

a. What is the decrease in the fish population in the first year? How many fish are left in the lake after one year? Show or explain your work.

   Decrease in fish population: ____________
   Number of fish left in lake: ____________
   Work or explanation:

b. Find the number of fish in the lake at the end of each of the next six years and record those numbers in the table below. Explain how you obtained your numbers.

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Fish</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3,000</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

   Explanation:

c. How long will it be before there are fewer than 1,000 fish in the lake?

d. The state wildlife management agency does not want the number of fish in the lake to decrease. So after seeing the above analysis, they decide to add 400 fish to the lake each year. Under these conditions how many fish will be in the lake at the end of the first year? Explain or show your work.

   Number of fish: ________________
   Explanation:

e. For the conditions described in Part d, use the words NOW and NEXT to write a rule that shows how to use the trout population in one year to estimate the trout population in the next year.

   NEXT = ________________, starting at ________________
3. Dirkson High School’s seniors would like to take a class trip to Washington, D.C. The tour company offers to arrange the entire trip for a set fee of $8,000. This cost is to be shared equally by the seniors who go on the trip. If \( n \) seniors go on the trip, you can calculate the cost \( c \) in dollars to each senior by the rule \( c = \frac{8,000}{n} \).

a. Use the rule to complete the table below.

<table>
<thead>
<tr>
<th>Number ( n )</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost per Senior ( c ) (in dollars)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. As accurately as possible, complete this sentence describing the pattern relating the number of seniors going on the trip and the cost per senior.

As the number of seniors going on the trip increases, …

c. Make a graph of the (number, cost per senior) relation.

![Graph of the (number, cost per senior) relation]

d. Explain how the shape of the graph above matches the description you wrote in Part b.

e. Find the number of seniors who would need to go on the trip in order for the cost per senior to be no more than $75. Explain how you determined your response.
4. Consider the four sketches of graphs below.

Match each table or rule with the most appropriate graph. You may use a graph more than once.

a. \( y = ax + b \)  
   Graph _____________

b. \[ \begin{array}{c|ccccccc} x & -3 & -2 & -1 & 1 & 2 & 3 \\ \hline y & -20 & -30 & -60 & 60 & 30 & 20 \end{array} \]  
   Graph _____________

c. \( y = ax^2 + b \)  
   Graph _____________

d. \[ \begin{array}{c|ccccccc} x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\ \hline y & -4 & 1 & 4 & 5 & 4 & 1 & -4 \end{array} \]  
   Graph _____________
UNIT 1 Patterns of Change

UNIT TEST

Form A
Suggested Solutions

1. a. 

<table>
<thead>
<tr>
<th>Figure Number $F$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Square Tiles $N$</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>

b. $NEXT = NOW + 1$, starting at 7
   
   One tile is added to the middle of each figure as you move from one figure to the next. Stage 0 has 7 tiles.

c. $N = F + 7$

2. a. Decrease in fish population: $(3,000)(0.90) = 300$ fish
   
   Fish left in lake: $3,000 - 300 = 2,700$ fish

b. 

<table>
<thead>
<tr>
<th>Year Number</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Fish</td>
<td>3,000</td>
<td>2,700</td>
<td>2,430</td>
<td>2,187</td>
<td>1,968</td>
<td>1,771</td>
<td>1,594</td>
</tr>
</tbody>
</table>

   Each year the number of fish is 90% of the previous number of fish. So, multiply the previous value by 0.90 to get the new value.

c. 11 years. At the end of 11 years, there will be approximately 941 fish in the lake.

d. Number of fish: $(0.9)(3,000) + 400 = 3,100$
   
   The number of fish will decrease by 10% and then increase by 400.

e. $NEXT = 0.90 \cdot NOW + 400$ or $NEXT = NOW - 0.1 \cdot NOW + 400$, starting at 3,000

3. a. 

<table>
<thead>
<tr>
<th>Number $n$</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost per Senior $c$ (in dollars)</td>
<td>400</td>
<td>200</td>
<td>133.33</td>
<td>100</td>
<td>80</td>
</tr>
</tbody>
</table>

b. As the number of seniors going on the trip increases, the price per senior decreases, rapidly at first and then more slowly. (Some students might notice that if you increase the number of seniors going on the trip by a factor of $k$, you reduce the cost per senior by a factor of $\frac{1}{k}$.)
c. Students may sketch either a scatterplot or a continuous graph.

d. Since the graph gets closer to the horizontal axis as you look from left to right, the cost per senior decreases as the number of seniors increases. This indicates that the price decreases more slowly as the number of seniors increases. (Additionally, students may notice that the vertical distance between adjacent, evenly spaced points on the graph decreases as the number of seniors increases.)

e. Students can zoom in on a calculator table of values or trace a calculator graph to determine that if 107 seniors go on the trip, the cost per senior will be $74.77. So, if 107 or more seniors go on the trip, the cost is less than $75 for each student.

4. a. Graph II
   b. Graph III
   c. Graph I
   d. Graph I
1. The Farm Bureau Feed Company tested a new weight-gain program for pigs. The company found that the weight of an average pig increased by 0.7 kg per day after going on the weight-gain program. Grunt is a pig that weighs 25 kg and is put on the weight-gain program.

a. Make a table of Grunt’s weight for each day of the first week of his weight-gain program.

<table>
<thead>
<tr>
<th>Day Number</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (in kg)</td>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Use NOW to stand for Grunt’s weight on any given day and NEXT to stand for his weight the following day. Write a rule that shows how to estimate Grunt’s weight on any given day.

*Rule: ________________________________*

c. Write a rule that shows how Grunt’s weight $W$ depends on the number of days $N$ he has been on the weight-gain program.

*Rule: ________________________________*

d. Describe as clearly as possible the pattern of change in weight as number of days increases.

*Description: ____________________________*

e. A pig must weigh 75 kg before it is sold. How long will Grunt be on the weight-gain program before he is sold? Explain your reasoning.

*Explanation: ____________________________*

f. Find the answer to Part e in a different way than you did in Part e.
2. The shelf life of milk is a function of the temperature at which the milk is stored. The following table gives the shelf life (in days) of milk stored at various temperatures (in degrees Fahrenheit).

<table>
<thead>
<tr>
<th>Storage Temperature $t$ (in °F)</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shelf Life of Milk $L$ (in days)</td>
<td>20</td>
<td>7.5</td>
<td>4.6</td>
<td>3.3</td>
</tr>
</tbody>
</table>

a. Write a sentence describing the way the shelf life of milk changes as temperature changes.

b. Which rule below best matches these data? Circle one.

\[ L = t - 15 \quad \quad L = \frac{4}{7} t \quad \quad L = \frac{60}{(t - 32)} \]

c. Use what you believe is the best rule to find to the nearest tenth of a day, the shelf life of milk stored at 42°F and at 55°F. Show or explain your work.

Shelf life at 42°F: ______________

Shelf life at 55°F: ______________

Explanation:

d. To the nearest tenth of a degree, at what temperature will the shelf life of milk be 10 days? Show or explain your work.

e. To the nearest tenth of a degree, for what temperatures will the shelf life of milk be less than 8 days? Show or explain your work.
3. Your older sister has just been hired to work at an accounting firm in your town. Her beginning pay is $12 per hour. When she was hired she agreed that for the first five years she would be given a 5% raise at the end of each year.

a. Complete the table below showing how hourly pay \( p \) in dollars depends on raises received \( n \).

<table>
<thead>
<tr>
<th>Raises Received ( n )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hourly Pay ( p ) (in dollars)</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Let \( \text{NOW} \) represent her pay in a given year and \( \text{NEXT} \) indicate her pay in the following year. Write a rule using \( \text{NOW} \) and \( \text{NEXT} \) that matches the pattern of change in the table above.

Rule: __________________________

c. If she continues to get a 5% raise each year, how many raises will she need to get before she is earning at least $20 per hour? Explain how you found your answer.

Raises received: ______________

Explanation:

d. Your older sister is trying to write a rule that would give hourly pay based on the number of years that she has been working at the accounting firm. She thinks the rule will have the form \( y = ax + b \). Do you agree or disagree with her? Explain why.
UNIT 1Patterns of Change

UNIT TEST

Form B
Suggested Solutions

1. a. | Day Number | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
    | Weight (in kg) | 25 | 25.7 | 26.4 | 27.1 | 27.8 | 28.5 | 29.2 | 29.9 |

b. $NEXT = NOW + 0.7$, starting at 25

c. $W = 25 + 0.7N$

d. As the number of days increases, the weight increases. The weight increases $0.76$ kg for each day.

e. After 72 days, Grunt will first weigh more than $75$ kilograms. Students can find this using their calculator and the table of values, a calculator-produced graph, recursively using their calculator, or symbolically if they learned that method previously.

f. Other methods should produce 72 days.

2. a. At $35^\circ F$, the shelf life is $20$ days. The shelf life then decreases as temperature increases. At first, the decrease is very rapid, but then it slows down, reaching just $3.3$ days at $50^\circ F$.

b. $L = \frac{60}{t - 32}$ is the best rule.

c. At $42^\circ F$, $L = \frac{60}{42 - 32}$; the shelf life is $6$ days.

At $55^\circ F$, $L = \frac{60}{55 - 32}$; the shelf life is about $2.6$ days.

d. The table suggests that the desired temperature is between $35^\circ F$ and $40^\circ F$. Guessing and testing, or observing that the denominator $t - 32$ must be $6$, gives the solution $38^\circ F$.

e. From the table or graph, it can be determined that for temperatures greater than $39.5^\circ F$, the shelf life of the milk will be less than $8$ days.

3. a. | Raises Received n | 1 | 2 | 3 | 4 | 5 |
    | Hourly Pay p | 12.60 | 13.23 | 13.89 | 14.59 | 15.32 |

b. $NEXT = 1.05 \cdot NOW$, starting at $12$

c. Eleven years. Start with $12$ and continue to multiply by $1.05$ until the salary first exceeds $20$. Keep track of the number of times you multiply by $1.05$ to first exceed $20$.

d. The sister is incorrect. From the table, you can see that each year the increase in pay is greater than it was the previous year. But in a table of values for a rule of the form $y = ax + b$, the amount of increase is always the same.
1. With the cost of college increasing every day, it is important that parents make plans about how they can help pay for their child’s college education. The descriptions below indicate savings plans used by two different families.

**The Ortegas**
Mr. and Mrs. Ortega put $1,000 into a savings account every year on their daughter’s birthday through her fifth birthday. They did not put any money into the savings account after that. The account earned 5% interest per year, which was paid on the last day of each year.

**The Thompsons**
Mr. and Mrs. Thompson could not afford to save when their son was young. They began putting $1,500 into a savings account on their son’s thirteenth birthday. They continued this through his eighteenth birthday. The account earned 5% interest per year, which was paid on the last day of each year.

Use a spreadsheet or calculator to help you determine which family had more money saved for college at the end of the year that their child turned 18. Which family made the better investment? Clearly show or explain your work.

2. The freshman class at King High School is having a car wash during the fall parent night. The students asked their parents how much they would be willing to pay to get their car washed while they were at the meeting. The responses are summarized in the table below.

<table>
<thead>
<tr>
<th>Cost of Car Wash (in dollars)</th>
<th>3.00</th>
<th>4.50</th>
<th>5.00</th>
<th>7.50</th>
<th>10.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Customers</td>
<td>400</td>
<td>325</td>
<td>300</td>
<td>175</td>
<td>50</td>
</tr>
</tbody>
</table>

What price should the students charge in order to maximize the income from the car wash? Clearly show your work and explain your reasoning. Your explanation should include any rules, tables, or graphs that you used.

3. Review the population studies presented in this unit. Choose a population that you would like to study. You might choose to study the population of a country or an animal species. Before you collect data, make a prediction about the pattern of change you expect to see in your population and give the reasons for your prediction. Reasons might include natural disasters or wars as possible examples. After making your conjecture, gather data from the library or other pertinent sources concerning changes over time for your population. Make tables, sketch graphs, and describe the trend, if any, that describes the data for your population. Compare your description to the conjecture you made before you gathered the data and use your work to make predictions about future growth of the population. Summarize your findings in a presentation or report. Be sure to discuss any “strange” fluctuations in the data.
These three assessment items may be used in a variety of ways. You may assign them to individuals, pairs, or groups of students. You may wish to use them in place of some part of an in-class end-of-unit test.

1. The spreadsheet on the following page can be used to explore this situation. The Ortegas invested $1,000 per year for five years for a total investment of $5,000. The Thompsons invested $1,500 per year for 6 years for a total investment of $9,000. However, the calculations show that the Ortegas will have slightly more money when their child turns 18. It seems that the Ortegas made the better investment. This problem shows the power of compound interest.

<table>
<thead>
<tr>
<th>Age of Child</th>
<th>Ortega</th>
<th>Thompson</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,000</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2,050</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>3,152.5</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>4,310.125</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>5,525.63125</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>5,801.91281</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>6,092.00845</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>6,396.60888</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>6,716.43932</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>7,052.26129</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>7,404.87435</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>7,775.11807</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>8,163.87397</td>
<td>1,500</td>
</tr>
<tr>
<td>14</td>
<td>8,572.06767</td>
<td>3,075</td>
</tr>
<tr>
<td>15</td>
<td>9,000.67105</td>
<td>4,728.75</td>
</tr>
<tr>
<td>16</td>
<td>9,450.70461</td>
<td>6,465.1875</td>
</tr>
<tr>
<td>17</td>
<td>9,923.23984</td>
<td>8,288.44688</td>
</tr>
<tr>
<td>18</td>
<td>10,419.4018</td>
<td>10,202.8692</td>
</tr>
</tbody>
</table>
2. The equation for the line that matches the \((\text{cost}, \text{customers})\) data is \(y = -50x + 550\). Thus, the income function is \(I = x(-50x + 550)\). The maximum income will be obtained when the price per car wash is $5.50 and the income will be $1,512.50.

Student work should clearly indicate how they arrived at each function and how they determined the maximum income. It should include symbolic rules, graphs, and tables of values that were used in solving the problem.

3. Judge your students’ work on the completeness and creativity of their work, their methods of collecting the data, their initial prediction, and their final analysis. The report should include detail about the procedures that they used and the discussion of the findings should include appropriate symbolic rules, graphs, and tables of values. Their conclusions should follow logically from procedures and results and should be presented in a clear and complete manner.
**UNIT 1  Patterns of Change**

**Name** ____________________________

**Date** ____________________________

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**PROJECT**

**Strength Testing**

**Purpose**

Mathematics can be used to analyze properties of many things in the world around us. During the unit, you explored relationships between many different pairs of variables. In this project, you will explore the strength and “rigidity” of different building materials.

**Directions**

1. Collect materials that you would like to test. You might choose to use thin strips of wood, metal, plastic, or some other building material. You should have at least five different strips. All your strips should use the same materials but should have different lengths, widths, or thicknesses. You will need to decide which characteristic—length, width, or thickness—you would like to explore.

2. Make a hypothesis about how you think the strength and rigidity of your material will vary among your strips.

3. Gather data about the strength and rigidity of the material. Begin by letting each strip be a bridge between two cement blocks or other hard flat surfaces. (Chairs, tables, or desks will work fine.) Now add weights to the middle of each strip until it breaks. Measure the “sag” as you add each weight. You will need to decide how best to make your measurements and how to record your data.

4. Analyze your data by exploring any relationships that are present and thinking about what they tell you about the strength of the material.

5. Write a brief report that states your hypothesis, describes your testing procedures and analysis, and states any conclusions you can make about the material that you tested. Your report should include a description of the materials used, sketches of your experiment, tables and graphs of the data, and any symbolic rules that you might have used in analyzing the data. You should also clearly explain the reasoning you used to draw your conclusions. Finally, you should compare your conclusions to your initial hypothesis.
UNIT 1  Patterns of Change

PROJECT

Suggested Solutions
Strength Testing

This project is probably best done by pairs or groups of students. You may want to have some materials that students may use to make their bridges. Students will need to make several decisions while they work on this project. They first need to decide on one characteristic to explore (length, width, or thickness), then they need to decide how they will add the weight, and lastly they will need to decide how to measure the “sag.” Depending on your students, you might need to help them with any one or all of these decisions.

Suggested Timeline
This project will probably require a week for students to complete. Be sure to check with students about the characteristic they are exploring and how they will gather the data. You will probably want to do this before the students spend time gathering their data. They should be able to gather their data, analyze it, and write up their findings in several days.

Report Format
Each report should contain the following:
• A statement of the hypothesis
• A description of the testing procedure
• An organized presentation of the data gathered in the experiment
• Tables, graphs, and symbolic rules that help illustrate the results of the experiment
• Explanation of the reasoning used to draw conclusions
• Conclusions of the experiment and a comparison of the conclusions to the initial hypothesis
UNIT 1 Patterns of Change

PROJECT

More Relationships

Purpose
A key goal for this unit was for you to recognize the rich variety in which quantities vary in relation to each other. During the unit, you worked with many such relationships. In this project, you will review the relationships from the unit and then explore other real-world relationships with the intent of furthering your understanding of relationships between two quantities.

Directions
1. In this unit, you analyzed many relationships between real-world quantities. Review your work from the unit and make a list of relationships that you explored.

2. For each relationship that you identify, describe each pair of variables and how they are related to each other. Identify the independent and dependent variables where appropriate. Then sketch a graph that would reflect the relationship. Label each graph so that the reader knows what variables are being used. In cases where there are logical dependent and independent variables, be sure to place the independent variable on the horizontal axis of your graph.

3. Make a list of ten relationships that are not discussed in this unit. Include relationships that will result in at least three different shapes of graphs. For each relationship, define as clearly as you can the pair of variables and how they are related to each other. Also sketch a graph that indicates the overall pattern that you would expect in the relationship. Explain why you drew the graph that you did.
This project could be completed by individuals or by pairs of students. If students work in pairs, they may want to review different lessons of the unit and compile the list of relationships in that lesson. The students can then merge their lists. You may also want to use this project as a way to review some of the big ideas of this unit.

**Suggested Timeline**
There should be two deadlines—one for compiling the lists of relationships found in the unit and one for submitting the student-generated relationships and their graphs. Provide students with feedback to the first part of the project before assigning the last part, to be sure that they understand what is meant by “relationships between real-world quantities.” A few days should probably be enough time for students to complete each part of the project.

**Solutions**
1–2. For each relationship, students should identify the independent and dependent variables (when appropriate), sketch and label a graph, and write a sentence describing how the quantities are related. Some of the real-world relationships that are found in the unit are listed in the table below. The table includes a brief description of the relationship. Students should not be expected to name the function type at this point but rather describe the relationship. In addition to the information in the table below, students should supply a sketch of a graph for each relationship.

### Relationships

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Dependent Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight of jumper</td>
<td>Stretched length of cord</td>
<td>As weight increases, the stretched length increases at a constant rate.</td>
</tr>
<tr>
<td>Price of a ticket</td>
<td>Number of tickets sold</td>
<td>As price increases, the number of tickets sold decreases at a constant rate.</td>
</tr>
<tr>
<td>Price of a ticket</td>
<td>Income</td>
<td>The income increases to a maximum and then decreases.</td>
</tr>
<tr>
<td>Play number of a game</td>
<td>Cumulative profit</td>
<td>There is not a predictable pattern in this relationship.</td>
</tr>
<tr>
<td>Average speed</td>
<td>Race time</td>
<td>As average speed increases, the race time decreases. It decreases quickly at first and then more slowly.</td>
</tr>
<tr>
<td>Number of days</td>
<td>Earnings</td>
<td>As the number of days increases the earnings increases at an increasing rate.</td>
</tr>
</tbody>
</table>
### Independent Variable | Dependent Variable | Dependent Variable
--- | --- | ---
Temperature forecast | Number of swimmers | Will depend on student assumptions
Weight | Postage charge | As the weight increases, the postage charge increases.
Weight | Distance from fulcrum | As the weight increases, the distance decreases at a decreasing rate.
Year | Population | Population generally grows at an increasing rate or decreases at a decreasing rate.
Time into jump | Height of bungee jumper | The jumper's height decreases, then increases, then decreases, then increases, …
Hours of flight | Fuel left in tank | As the hours of flight increase, the amount of fuel left in the tank decreases at a constant rate.

3. Students should identify and describe relationships other than those explored in the unit.
PRACTICING FOR STANDARDIZED TESTS

Practice Set

Solve each problem. Then record the letter that corresponds to the correct answer.

1. If $N$ is an odd integer, which of the following numbers is also an odd integer?
   (a) $N \times N$  
   (b) $N + N$  
   (c) $3N - 1$  
   (d) $N - 1$  
   (e) $N + 5$

2. A T-shirt sells for $18 in a retail store. If this price is 120% of the wholesale price, what is the wholesale price?
   (a) $14.40$  
   (b) $15.00$  
   (c) $16.00$  
   (d) $16.20$  
   (e) $21.60$

3. Which of the following figures is the result of a half-turn about point $T$ of the figure at the right?

   (a)  
   (b)  
   (c)  
   (d)  
   (e) 

4. The population of Country X is growing at 3% per year. Which of the following does not correctly indicate the change in population from one year to the next.
   (a) $\text{NEXT} = 1.03 \cdot \text{NOW}$  
   (b) $\text{NEXT} = \text{NOW} + 0.03 \cdot \text{NOW}$  
   (c) $\text{NEXT} = 0.03 \cdot \text{NOW}$  
   (d) $\text{NEXT} = \text{NOW} + 0.05 \cdot \text{NOW} - 0.02 \cdot \text{NOW}$

5. In a quadrilateral, two of the angles have a measure of $90^\circ$ each. The measure of a third angle is $100^\circ$. What is the measure of the remaining angle?
   (a) $70^\circ$  
   (b) $80^\circ$  
   (c) $170^\circ$  
   (d) $190^\circ$  
   (e) $280^\circ$

6. If $a > 0$ and $b < 0$, which of the following must be negative?
   
   I. $ab$  
   II. $\frac{a}{b}$  
   III. $a - b$
   
   (a) I only  
   (b) II only  
   (c) III only  
   (d) I and II  
   (e) All of them
7. Which graph matches the rule \( y = \frac{a}{x} \), with \( a > 0 \)?

(a) \( \text{Graph A} \)  
(b) \( \text{Graph B} \)  
(c) \( \text{Graph C} \)  
(d) \( \text{Graph D} \)  
(e) \( \text{Graph E} \)

8. Jane bought some peppermint patties. She gave half of them to her brother and then a third of those left to her sister. Now she has 6. How many peppermint patties did she buy?

(a) 18  
(b) 24  
(c) 30  
(d) 36  
(e) 42

9. \( \overline{AB} \), \( \overline{CF} \), and \( \overline{DF} \) intersect at point \( F \) and the measure of \( \angle AFC \) is \( 90^\circ \). The measure of \( \angle BFD \) is twice as much as the measure of \( \angle CFD \). What is the measure of \( \angle CFD \)?

(a) 15°  
(b) 22.5°  
(c) 30°  
(d) 45°  
(e) 60°

10. Which fraction has the greatest value?

(a) \( \frac{5}{17} \)  
(b) \( \frac{5}{15} \)  
(c) \( \frac{5}{13} \)  
(d) \( \frac{5}{11} \)  
(e) \( \frac{5}{9} \)

**Test-Taking Tip**

Test general properties of numbers by using specific numbers.

**Example** Look back at Item 1. To use this strategy, choose a specific odd number such as 3 to substitute for \( N \) in each of the listed expressions.

For choice (a): \( 3 \times 3 = 9 \times 9 \) is an odd integer.

For choice (b): \( 3 + 3 = 6 \times 6 \) is not an odd integer.

For choice (c): \( 3(3) - 1 = 8 \times 8 \) is not an odd integer.

Explain why choices (d) and (e) are not correct choices. So, the answer is (a).

Find, if possible, another test item in the practice set for which this strategy might be helpful. Try it.