Evaluating a Limit Algebraically

The value of a LIMIT is most easily found by examining the graph of f(x). However, the graph is not always given, nor is it easy to sketch. A limit can be evaluated "mechanically" by using one or more of the following techniques.

Direct Substitution

To evaluate \( \lim_{x \to a} f(x) \), substitute \( x = a \) into the function. If this results in a real value, this value is the limit.

Example: \( \lim_{x \to 2} (2x^2 + 3x - 5) = 2(2)^2 + 3(2) - 5 = 9 \)

Example: \( \lim_{x \to 2} \left( \frac{x^2 - 4}{x - 2} \right) = \lim_{x \to 2} \left( \frac{(x + 2)(x - 2)}{x - 2} \right) = 0 \) (this value is indeterminate)

**** The method of DIRECT SUBSTITUTION does not work.****

Algebraic Simplification

If \( f(x) \) and \( f(x) \) are equivalent algebraically, then

\[ \lim_{x \to 2} \left( \frac{x^2 - 4}{x - 2} \right) = \lim_{x \to 2} (x + 2) = 2 + 2 = 4 \]

because \( \frac{x^2 - 4}{x - 2} = \frac{(x + 2)(x - 2)}{(x - 2)} = x + 2 \)

In this second method, we try to "reduce" the function algebraically, and then use direct substitution.

NOTE: \( x^3 - y^3 = (x-y)(x^2 + xy + y^2) \) Special
\( x^3 + y^3 = (x+y)(x^2 - xy + y^2) \) Factoring Rules
Exercises: Evaluate each limit by direct substitution and/or algebraic simplification. If neither method produces a result, write NO LIMIT.

1) \( \lim_{x \to 3} x^2 = \)

2) \( \lim_{x \to 5} \left( \frac{x^2 + 25}{x + 5} \right) = \)

3) \( \lim_{x \to -4} \left( \frac{6x}{x - 2} \right) = \)

4) \( \lim_{x \to 0} \left( \frac{1}{x} \right) = \)

5) \( \lim_{x \to 5} \left( \frac{x^2 - 25}{x - 5} \right) = \)

6) \( \lim_{x \to 6} \left( \frac{x^2 - 25}{x - 5} \right) = \)

7) \( \lim_{x \to 2} \left( \frac{x^3 - 8}{x - 2} \right) = \)

8) \( \lim_{x \to 3} \left( \frac{x^3 + 27}{x + 3} \right) = \)

9) \( \lim_{x \to 1} \left( \frac{2x^2 + 3x + 1}{x + 1} \right) = \)

10) \( \lim_{x \to -1} \left( \frac{x + 1}{2x^2 + 3x + 1} \right) = \)

11) \( \lim_{x \to -1/2} \left( \frac{x + 1}{2x^2 + 3x + 1} \right) = \)

12) \( \lim_{x \to 0} \left( \frac{x^2 + 25}{x + 5} \right) = \)